

The incircle of a triangle

Topic: Geometry

Theme: Illustration of the connection of the radius of a circle inside a triangle, semi perimeter of the triangle and the triangle's area.

Abilities: Build geometrical figures by following instructions.

Material: paper, pencil, scissors

Level: Age 13-14

Remarkable points in a triangle

The *Incircle of a triangle* is also known as «*inscribed circle*». It is the largest circle that will fit inside the triangle. Each of the triangle's three sides is a tangent to the circle.

The incenter may be defined as the point where the internal angle bisectors of the triangle cross, as the point equidistant from the triangle's sides, as the junction point of the medial axis and innermost point of the grassfire transform of the triangle, and as the center point of the inscribed circle of the triangle.

Together with the centroid, circumcenter, and orthocenter, it is one of the four triangle centers known to the ancient Greeks, and the only one that does not in general lie on the Euler line.

Why is the luggage compartment of a tour coach at the bottom of the coach ?

Explanation: Lower the centroid (or center of mass) of a system is more stable the system will be. For the same reason extra passengers are not allowed on the upper deck of a crowded double-decker bus.

Why do you think racing cars are built low and broad?

Again for the same reason, lower the centroid is more stable the system will be. Also with a lower center of mass the driver can also

take sharp turns at high speed without risk of toppling.

Assume you are a city planner and you have pulled funds from 3 cities Delhi, Noida and Gurgaon to start a recreational club.

Now the question is at which place should I open it so that it is equidistant from all the three cities.

Explanation: Mark all the three cities on the map. Join them to get a triangle and find its circumcircle, because that is the place which is equidistant from all the 3 cities.

Connection between the radius of the circle inside the triangle, its semi perimeter and the area of the triangle.

The incircle or inscribed circle of a triangle is the largest circle contained in the triangle. It is tangent to the three sides.

The **three** angle **bisectors** of the angles of a **triangle** meet in a single point, called the incenter. In Euclid's *Elements*, Proposition 4 of Book IV proves that this point is the center of the inscribed circle (also called the incenter) of the triangle.

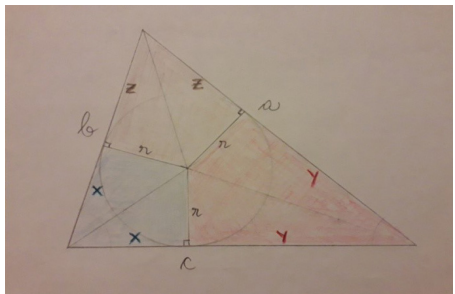
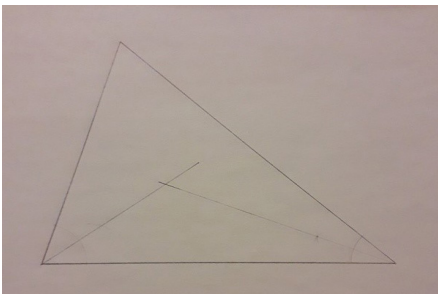
In any triangle, there is the equality: $r = S : p$

Where r is the radius of the circle inside the triangle, S is the area of the triangle and p is the semi perimeter of the triangle.



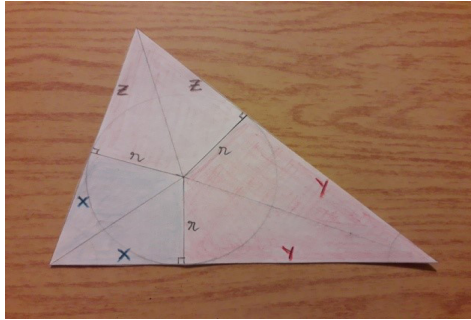
Take a piece of paper and draw a triangle. Draw two bisectors, then draw the incircle. From the centre of the circle, draw the tangential points.

$$a+b+c=2(x+y+z), \text{ and you get } p=x+y+z$$



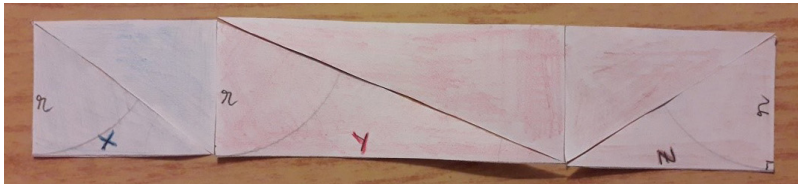
2

Cut the triangle in the 6 newly resulted triangles.



3

Regroup the triangles in order to form a rectangle with a length side r and the other one of length $x+y+z$.



4

The area of the initial triangle is equal to the one of the rectangle, so $S = r(x+y+z)$, $S = rp$.