



# System of linear equations

**Topic :** Algebra

**Theme :** Linear System of Equations

**Abilities :** Practice, through a gamified method, on how to find the solution for a linear system of equations, utilizing the method of substitution

**Material :** No extra material is needed

**Level :** 13-15 (second and third year of High School)

In mathematics, a system of linear equations is being composed of two or more linear equations which employ the same set of variables. A solution of a linear system is one assignment of values which satisfy all the equations of the system at the same time. For instance, for the following linear system of two equations in two variables  $x, y$ :

$$x + 2y = 7$$

$$x - y = 1$$

The solution is given by the assignment  $x = 3$  and  $y = 2$ , inasmuch such assignment of values makes both equations valid at the same time.

The same applies for a linear system of three equations in three variables  $x, y, z$ , such as the following:

$$x + 2y + z = 9$$

$$x - y - 2z = -3$$

$$x + y + z = 6$$

In which  $x = 2, y = 3$  and  $z = 1$  or  $(x, y, z) = (2, 3, 1)$  is the solution of this linear system.

Within the current tool, we will focus on the method of substitution, as a method of resolving a linear system of equations. We will try to explain the method through the following example, a linear system which involves 2 equations in 2 variables:

$$2x + 3y = 8$$

$$4x - 5y = -6$$

As a first step, we solve one of the two equations for  $x$  in terms of  $y$ , or for  $y$  in terms of  $x$ . In this case we choose to solve the first equation for  $x$  in terms of  $y$ :

$$2x + 3y = 8$$

$$2x = 8 - 3y$$

$$x = \frac{(8-3y)}{2}$$

$$x = 4 - \frac{3y}{2}$$

At this stage, we substitute this expression of  $x$  into the other (second) equation of the linear system. Hence the equation  $4x - 5y = -6$  will take the form of:  $4\left(4 - \frac{3y}{2}\right) - 5y = -6$

$$16 - 6y - 5y = -6$$

$$16 - 11y = -6$$

$$\frac{-11y}{-11} = \frac{-22}{-11}$$

$$y = 2$$

Now, we substitute  $y=2$  back into any equation of the system that involves the variable  $x$ . For instance, we substitute  $y=2$  into the equation  $2x + 3y = 8$ :

$$2x + 3 \cdot 2 = 8$$

$$2x + 6 = 8$$

$$2x = 2$$

$x = 1$ . So the solution is  $x = 1$  and  $y = 2$  or  $(x,y) = (1,2)$

## Task

Using the method of substitution presented above, try to find an arithmetic value for all the items contained within the following system of 8 equations:

$$2 \text{ t-shirts} + 1 \text{ skirt} = 1 \text{ butterfly}$$

$$1 \text{ high-heeled shoe} + 2 \text{ skirts} = 1 \text{ butterfly} + 1 \text{ ladybug}$$

$$1 \text{ t-shirt} + 2 \text{ dresses} = 1 \text{ bee} + 1 \text{ butterfly} + 1 \text{ caterpillar}$$

$$1 \text{ t-shirt} + 1 \text{ skirt} = 1 \text{ bee} + 1 \text{ ladybug} + 1 \text{ caterpillar}$$

$$1 \text{ butterfly} = 10 \text{ caterpillars}$$

$$1 \text{ ladybug} = 4 \text{ caterpillars}$$

$$1 \text{ bee} = 2 \text{ caterpillars}$$

$$1 \text{ caterpillar} = 5$$