15

Special products

Topic : Algebra
Theme : Visualize the factors of algebraic special products, by cutting forms and figures
Abilities : Draw, manipulate, realize geometric figures. Apply instructions
Material : Paper; ruler; colored markers; scissors; pay dough; knife..
Level : Age 14/15

Special products are common useful formulas, which can serve for simplifying calculations, solving equations, etc.

Examples:

$(a+b)(a-b)=a^2-b^2$...add times subtract
$(a+b)^2=a^2+b^2+2ab$ ....square of a binomial
$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$..........square of a trinomial
$(a+b)^3=a^3+b^3+3a^2 b+3ab^2$.....................cube of a binomial
$(a - b)(a^2\ ab + b^2) = a^3 - b^3$ ....difference of two cubes

These products are called «special» because they are very common, and they can be also very useful. Once one has recognized them, he (or she) can easily minimize the calculations. It is very important not to forget to calculate the terms $2ab$ (double product) and $3a^2b/3ab^2$ (triple products)

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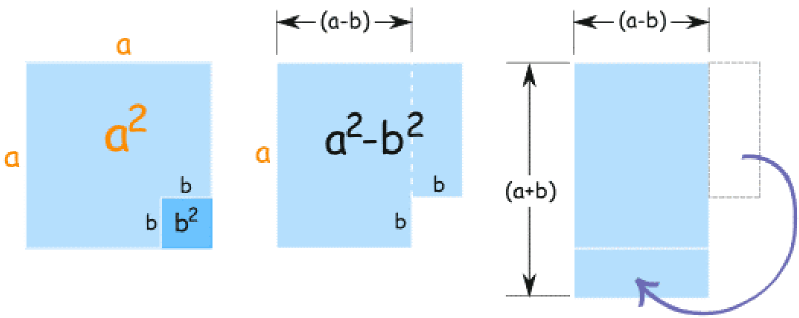
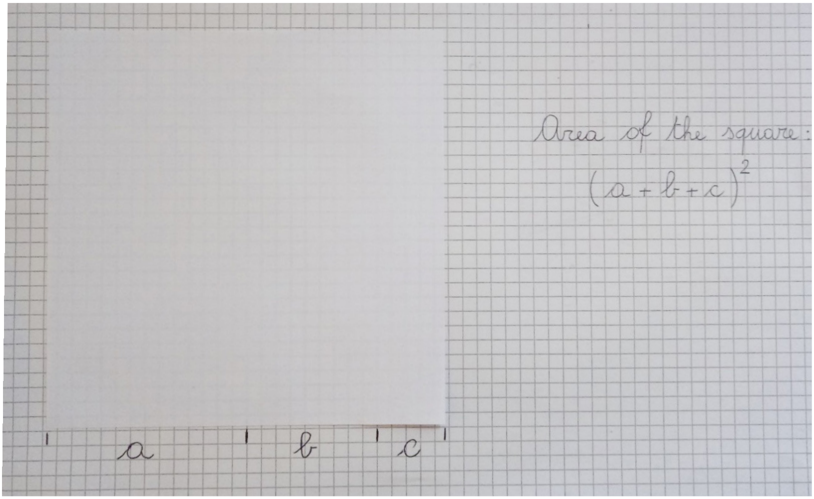
Add times subtract

- Draw a square of side *a*. Draw inside it, in the lower corner, a square of side *b*, as in the first figure below.
- Cut the smaller square from the bigger square. Then cut the rectangle on the right side of the figure, whose sides are *b* and (*a* - *b*), as in the second figure below.
- Put the smaller rectangle below the bigger rectangle, as in the third figure below.
- The second and third figure have the same area. The first area is $a^2 – b^2$; the second area is $(a + b)(a - b)$.

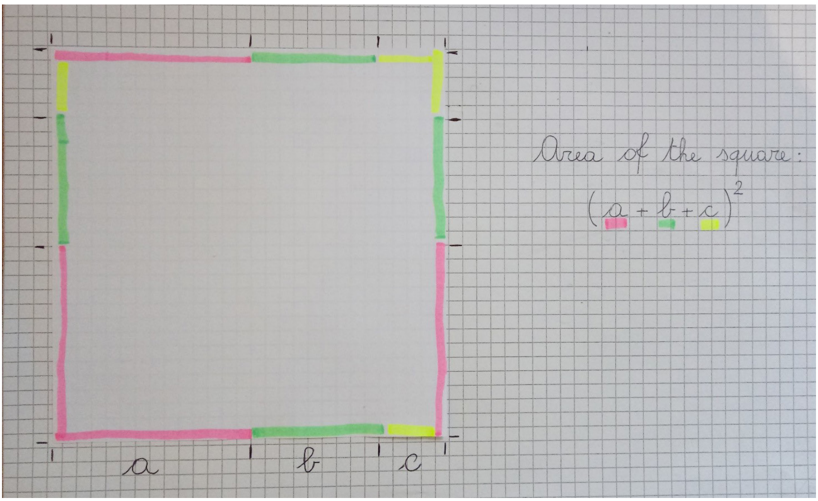


Square of a trinomial

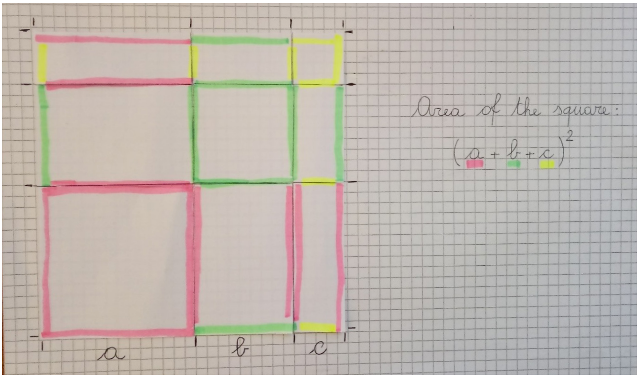
- Cut a white square and divide its side in three parts *a*, *b* and *c* (see Fig. 1)



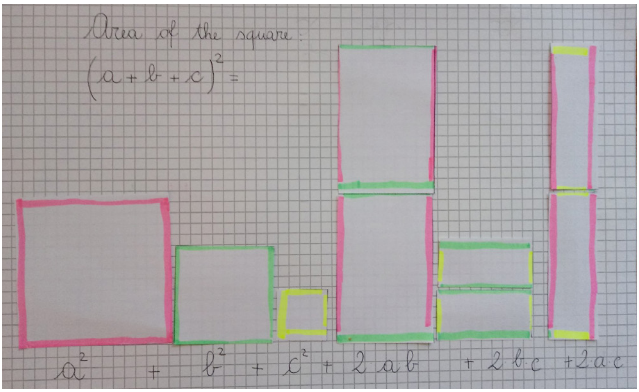
Divide all sides of the square in parts *a*, *b* and *c* and mark them with different colors (see Fig. 2)



Divide the great square in smaller squares and rectangles, using the coloured segments on its sides (see Fig. 3)



Cut all the parts of the square (two smaller squares and three couples of different rectangles) and write their areas. The sum of these areas equals the area of the white square (see Fig. 4)



Square of a binomial

Same procedure as square of a trinomial, but dividing the side of the square in two parts *a* and *b*.

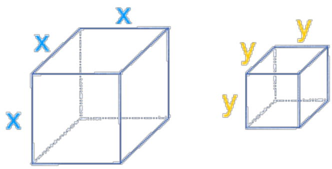
Cube of a binomial

- Prepare a cube made of play dough.
- Divide each side in two parts, by marking it with a little notch.
- Cut the cube in slices with a knife, following the notches.
- You will obtain two cubes, three identical prisms, and other three identical prisms.

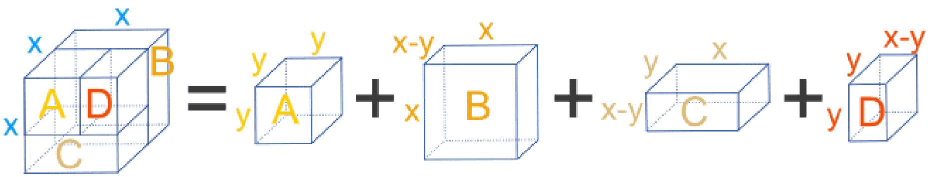
You can find the demonstration of the activity at the following link: https://youtu.be/rXoPaRDYNTQ

Difference of two cubes

Take two cubes of lengths x and y, as in the figure below :



The larger «x» cube can be split into four smaller boxes (cuboids), with box A being a cube of size «*y*», as in the figure below :



volumes of these boxes are:

- $A = y^3$
- $B = x^2(x - y)$
- $C = xy(x - y)$
- $D = y^2(x - y)$

Together, A, B, C and D make up the larger cube that has volume x^3:

$$x^3 = y^3 + x^2(x - y) + xy(x - y) + y^2(x - y)$$

$$x^3 - y^3 = x^2(x - y) + xy(x - y) + y^2(x - y)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$