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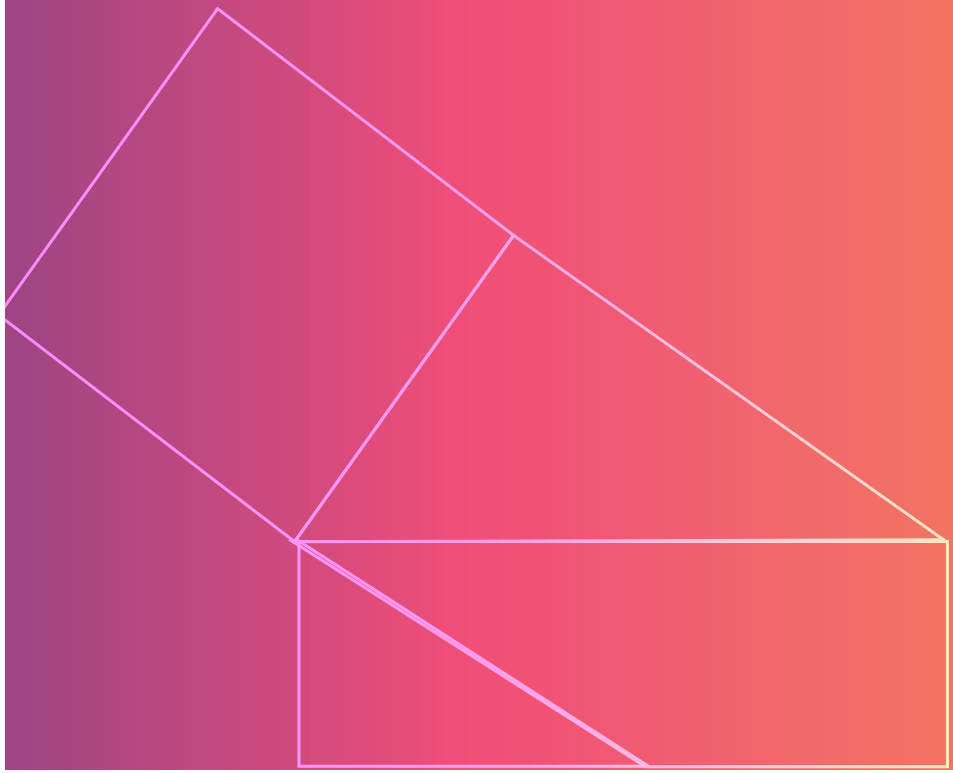


Right triangle theorems

Topic: Geometry
Theme: Illustrate by cutting the *Euclid's theorem*
Abilities: Visualize a theorem. Realize, manipulate geometric figures.
Material: Colored cardstocks; triangle rulers; scissors
Level: Age 15/16

Math Reality

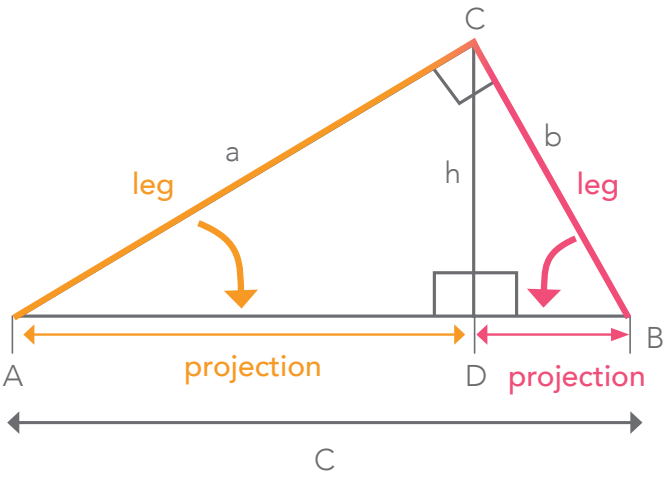
The right triangle theorems relate legs, hypotenuse, altitude and projections of a right-angled triangle.

These theorems are two mathematical rules, named leg rule and altitude rule, that define the relation between different parts of a right-angled triangle. In Italy these theorems are known respectively as First and Second *Euclid's Theorem*. Together with *Pythagoras'sTheorem*, they are the basis of geometry concerning triangles.

Both of them can be derived from *Euclid's* Elements (Proposition 8, Book VI). They can be stated in two different ways depending on which property one wants to highlight:

1. by equivalent figures or
2. by a relation between segment lengths.

In secondary school they are generally used as relations between segments. But the demonstration is much easier using equivalent figures.



Following the notation in the above figure, the First *Euclid Theorem* (leg rule) can be expressed as:

$AC^2 = AB \times AD$ for leg AC , and

$BC^2 = AB \times DB$ for leg BC

While the Second *Euclid Theorem* (altitude rule) can be expressed as:

$CD^2 = AD \times DB$

The two *Euclid's Theorems*

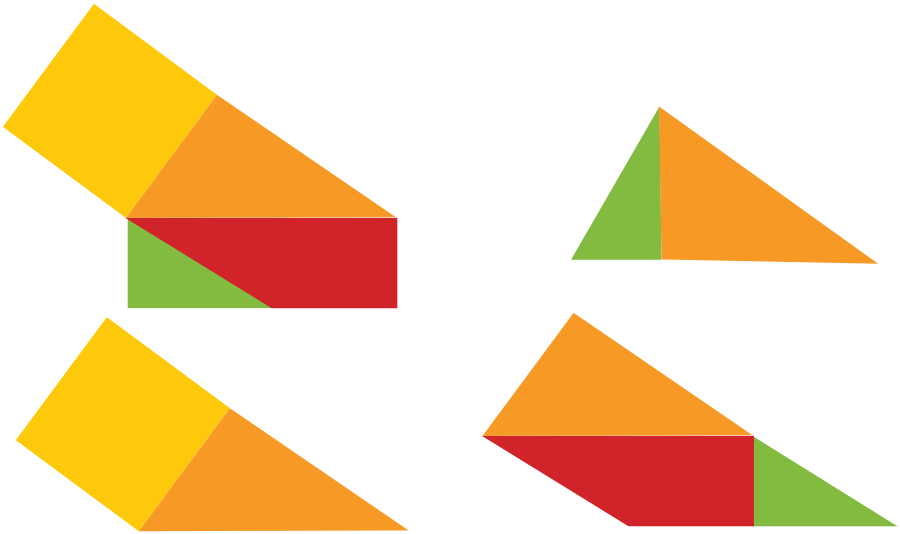
You can find a demonstration of this activity at the following link:
https://www.youtube.com/watch?v=eC5WwbmOu2U&t=44s

Prepare the basic pieces needed for the demonstration:

- Two right angled triangles (orange forms) with sides named as the previous figure. We will call a the leg opposite to angle α; b the leg opposite to the angle β; c the hypotenuse; h the height; a' and b' the projection of legs a and b respectively.
- Same triangle as above, cut in two parts by its altitude (green and light blue triangles)
- A square with side b (yellow one) and a square with side h (blue one)
- A rectangle with sides a' and b' (red one)
- a trapezoid with bases (c - h) and c, and height b'

First *Euclid's Theorem* demonstration

As shown in the figure below, the yellow square is equivalent to the red trapezoid plus the green triangle:



Second *Euclid's Theorem* demonstration

As shown in the figure below, the blue square is equivalent to the red rectangle