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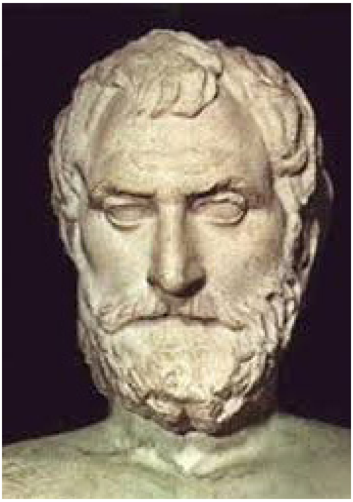
Thales' Theorem

Topic : Geometry
Theme : Thales' theorem and "similar triangles"
Abilities : Being able to : comprehend the Thales' theorem / apply one of the criteria of similar triangles based on one problem that has been drawn from the history of mathematics
Material : No extra materials are needed for the conduction of this exercise
Level : Age 14-15

Math Reality

Who was Thales of Miletus?



Thales of Miletus was born around 652BC in Miletus, Greece.
He is considered as the premier pre-Socratic philosopher, the first of the seven sages of antiquity. He was mathematician, physicist, astronomer, engineer, meteorologist. He was the founder of the Ionian School of natural philosophy in Miletus.

Aristotle and other ancient philosophers considered Thales as the first Greek philosopher; Thales was the one who achieved to approach and explain the natural phenomena through scientific logic, refusing to accept any previous interpretations of natural phenomena, which until then had been based solely on myths, legends and religious beliefs. Hence, Thales of Miletus has been considered as the one who first paved the way for scientific thought.

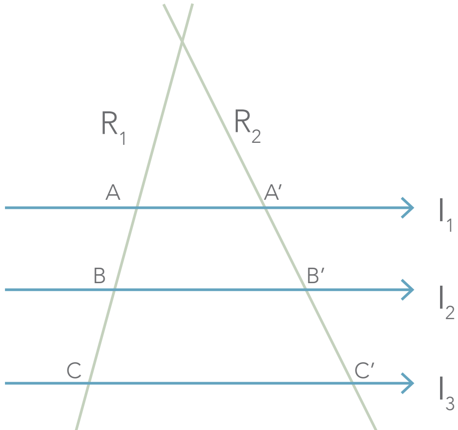
Thales' theorem

Thales of Miletus is widely known for the theorems attributed to him in the field of geometry. One of them is the theorem presented below:

If three parallel straight lines l_1, l_2 and l_3 cut (intersect) other two ones, namely R_1 and R_2, then they produce proportional segments

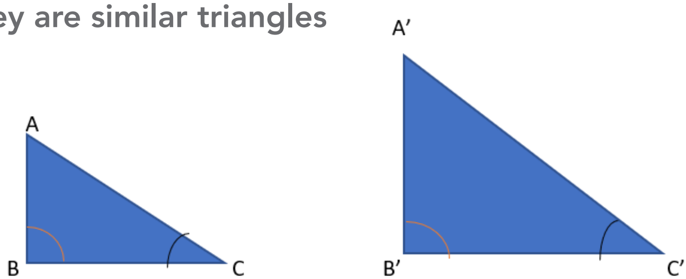
That is, if $l_1 // l_2 // l_3$ intersect R_1 and R_2, then

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$$



Moreover, the theory of similar triangles is strongly correlated with Thales' theorem. Specifically, there are three criteria of similarity; here we will focus on the second criterion of similarity (usually found as AA criterion of similarity, which is being formed as follows:

If two triangles have two of their angles equal (one by one) then they are similar triangles



Let's suppose that angle B of triangle ABC is equal to angle B′ of A′B′C′ and that angle C is equal to C′. Then, according to the AA criterion of similarity given above, we can conclude that the triangles ABC and A′B′C′ are similar, thus getting the following proportion:

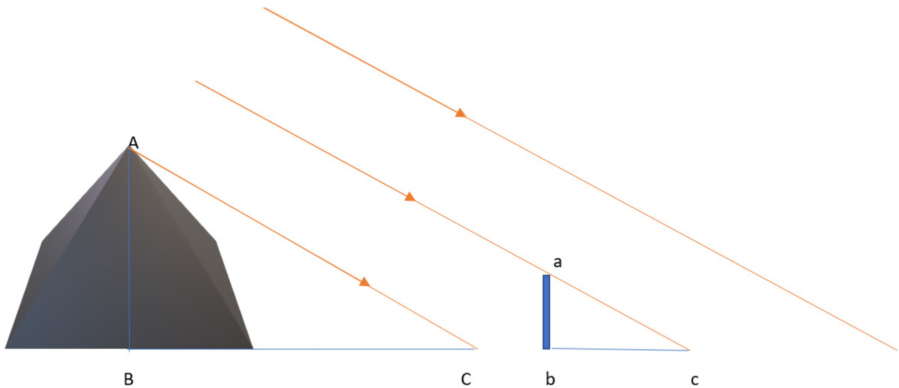
$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'} = \lambda$$ **where λ is called "similarity ratio "**

Task

Based on the history of mathematics, and according to Plutarch (Essayist), Thales of Miletus used the theory of similar triangles in order to solve a practical problem which had arisen at that time. It is said that since then, nobody had managed to calculate the height of the pyramid of Cheops, due to the peculiarities of its shape (it had been built sideways).

However, Thales managed to solve this problem by calculating the length of the shadow of the pyramid, thus earning the admiration of the Egyptian King, Amasis.

The following picture depicts Thales's solution :



At a particular time of the day during which the rays of the sun were sideways of the pyramid, Thales placed a stick in parallel with the pyramid, whilst he immediately observed the shadow of the stick on the ground. Subsequently, he realized that the length of the stick (ab), the length of the stick's shadow (bc), as well as the length of the pyramid's shadow (BC) were all easily measurable quantities. Accordingly, he managed to count the pyramid's height by applying the first criterion of congruence in the two triangles that had been shaped.

Observe the picture above and work on the following questions:

Question 1: Which two triangles did he use in order to apply the AA criterion of similarity? Use the letters given in the picture above to define the triangles.

Question 2: How did Thales of Miletus prove that he could apply the specific criterion of similarity? In other words, how did he know that the prerequisites stated in the AA criterion of similarity were valid for this specific case?

Question 3: Which is the proportion that Thales formed in order to estimate the height of Cheops's pyramid?

Question 4: Let's suppose that the length of the stick was 2 feet, the length of its shadow was 4 feet, while the length of the pyramid's shadow was 912 feet. By applying the proportion of 'Question 3', calculate the height of the Cheops's pyramid.

Question 5: Calculate the similarity ratio.