LESSON SCENARIO 02: COSINE RULE

Topic: Trigonometry

Level: Age 17 -18

Foreknowledge: Concept of sine and cosine, area of a quadrangle

Correlation: Physics (operation with vectors), Astronomy (parallax method)

LEARNING OUTCOMES

- Use trigonometry
- Find out Cosine Rule
- Be able to compare different statements of the same concept

TEACHING METHODS

- Practical work
- Hands-on activity
- Group work

KEY WORDS

- Acute angle
- Obtuse angle
- Triangles
- Quadrangles
- Equivalence of polygons

RESOURCES

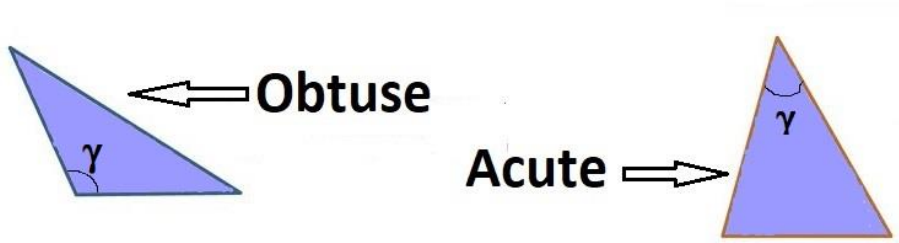
- Cardstock
- Triangle rulers
- Pencil
- Coloured pens
- Scissors
- Two cardboard reference triangles

ACTIVITIES

INTRODUCTION TO THE LESSON (5 minutes)

The teacher asks the students to list some definitions and formulas: definition of cosine of an angle; formula for determining the area of a square; formula for determining the area of a parallelogram. They will use these formulas during the activity.

Then he/she divides the class in four groups and gives to each group a violet cardboard triangle: two of them are acute angled triangles, the other two are obtuse angled triangles (see figure):

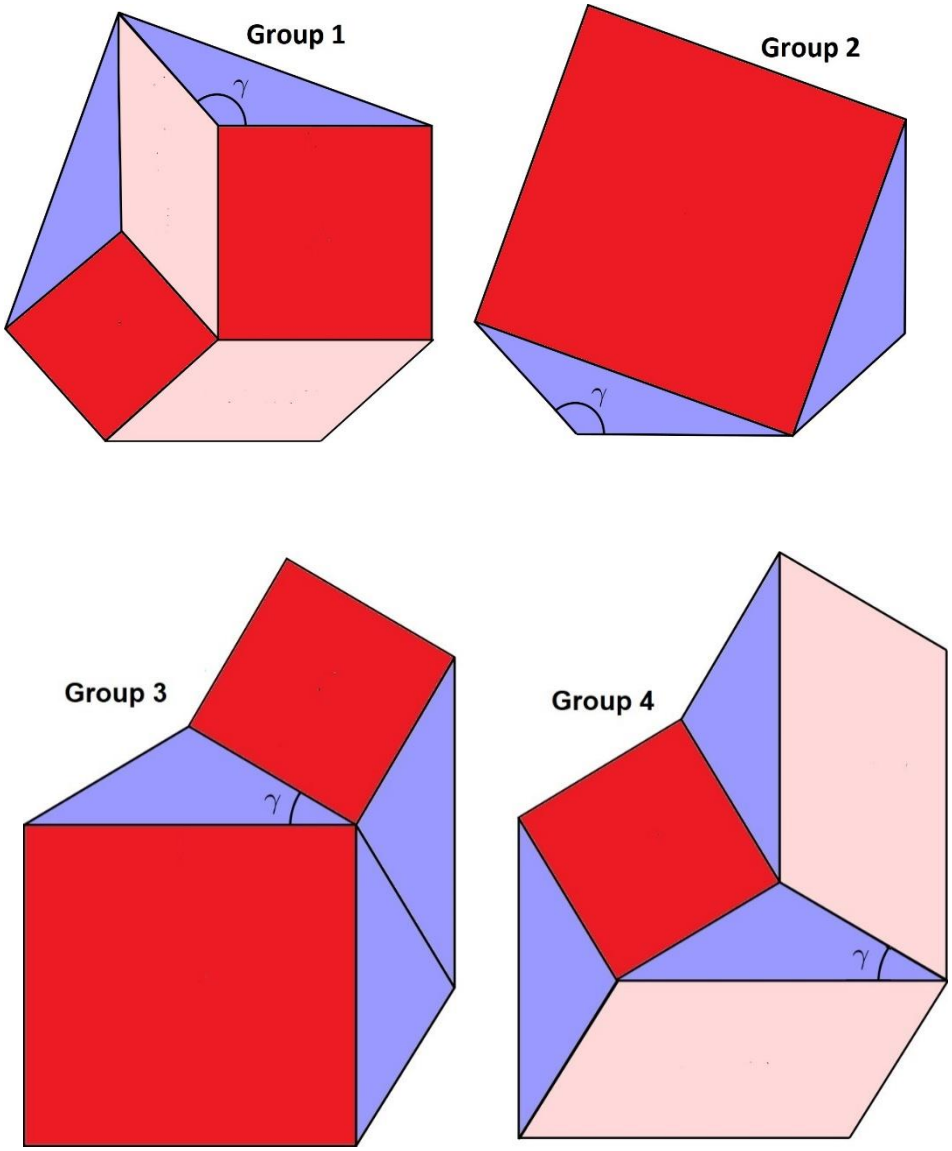


The teacher asks to name one angle of the triangle (γ as in figure above) and to name the three sides of the triangle a, b and c (where c is the side opposite to γ)

Then he/she says to the students that they are going to get the Cosine rule by comparing figures.

HANDS-ON PART OF THE LESSON (10 - 15 minutes)

The teacher asks each group to draw one of the following figure, using as a starting point the triangle they have just received. The students can use cardstock, rulers and scissors. They can also colour their figure, using the colours below.



An example of how to conduct the drawing:

Group 1: draw the violet triangle on the cardboard. Draw a red square below the lower side of the triangle (adjacent toγ). Draw a pink parallelogram starting from the left side of the square. Etc.

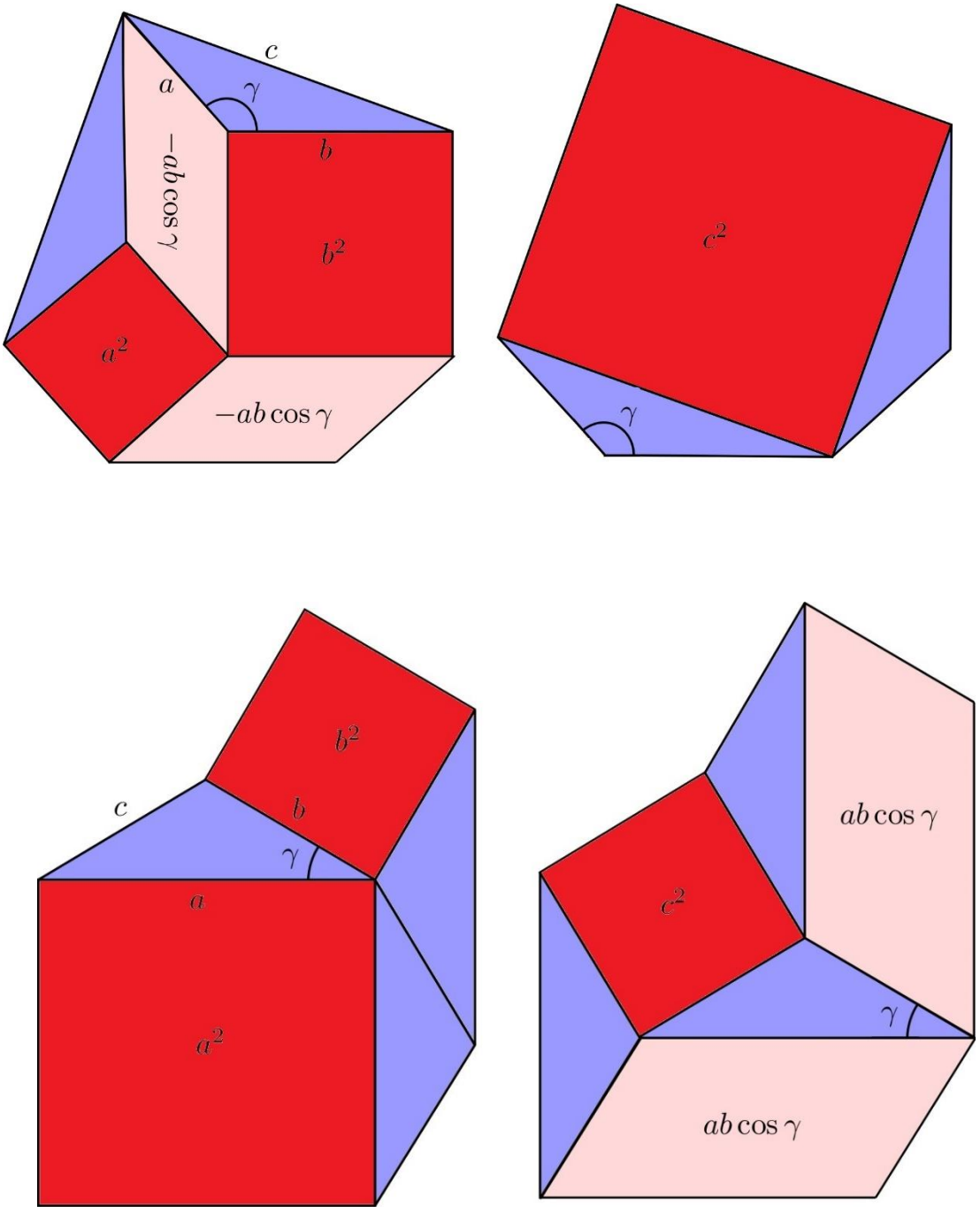
Group 2: draw a red square of side c, the longest side of the triangle (opposite toγ). Draw a violet triangle on one side of the square. Etc.

Group 3: draw the violet triangle on the cardboard. Draw two red squares starting from the two sides of the triangle adjacent toγ (a and c). Etc.

Group 4: Draw a red square with side c. Draw the violet triangle on three of the four sides of the red square. Etc.

CALCULATION PART (10 - 15 minutes)

Each group have to calculate the area of its figure, by adding the areas of various parts (squares and parallelograms). They should come to this calculation (see figures).



CONCLUSION (10 - 15 minutes)

From the equivalence of the figures (two by two) and some algebraic arrangements, the students will arrive to the Cosine rule formula:

$$c^2 = a^2 + b^2 - 2ab \cdot cos\gamma$$

An example of algebraic arrangements for groups 3 and 4:

The area of figure 3 (on the left above) is

$a^2 + b^2 + A_T + A_T + A_T$ where A_T is the area of the violet triangle.

The area of figure 2 (on the right above) is $c^2 + A_T + A_T + A_T + abcos\gamma + abcos\gamma$.

Equalling the two areas we get: $a^2 + b^2 + 3A_T = c^2 + 3A_T + 2abcos\gamma$.

By subtracting $(3A_T + 2abcos\gamma)$ from both sides of the equality we get the final formula.

Similar procedure for groups 1 and 2.

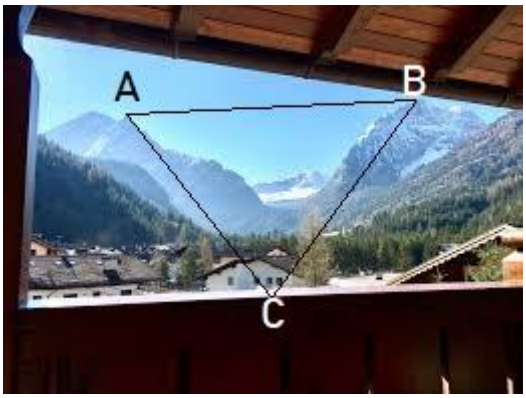
The teacher helps the students to write down the statement of the Cosine rule: "The square of the length of any side of a triangle equals the sum of the squares of the length of the other sides minus twice their product multiplied by the cosine of their included angle."

1. DO I KNOW THE COSINE RULE?

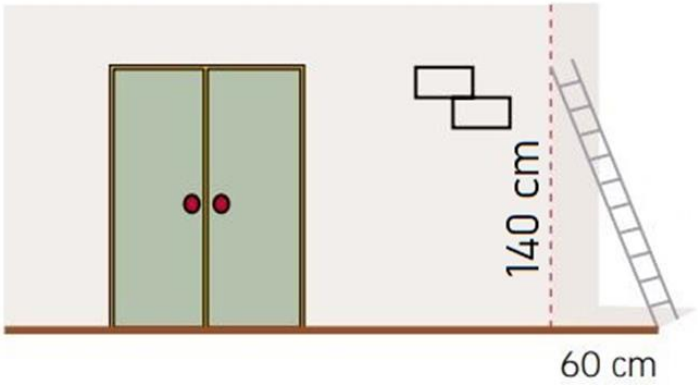
2. CAN I APPLY IT?

The students must answer to these questions in 5-10 minutes:

1) A climber C gazes at two peaks A and B, which are respectively 8 km and 12 km away as the crow flies from the hut window. If the climber has to range 75 degrees from one peak to another, what is the distance between the two peaks?



2) In the figure, a ladder is shown leaning against a wall. Find the length of the ladder. How does the Cosine rule become in this special case? Can you find out the angle made with the floor?



Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- **When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.**
- **You can use checklists for your students to make sure they have done all the steps**
- **Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.**
- **At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process**
- **Make sure that the material the students manipulate is easy enough to grasp**
- **While using different media (paper, computer and visual aids) choose different background than white which can be too bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colours to learn more about student's preference.**
- **To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.**

EXAMPLE:

1. Start every lesson with a short "CHECK-IN"

- **Today, we will study the topic (name of the topic)**
- **I will tell you about: (name 3 keywords connected with the topic)**
- **Then I will present exercises: (name the exercises from the student book)**
- **Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)**
- **Once the exercises will be done [To continue]**

2. Then finish lesson with a short "CHECK-OUT"

- **During the lesson we learn about (topic of the lesson)**
- **The most important things were: (name 3 keywords connected with the topic)**
- **We were able to do… (tell about the work student done during the lesson)**
- **We will explore the topic next time when we will learn about (name the following topic)**

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.