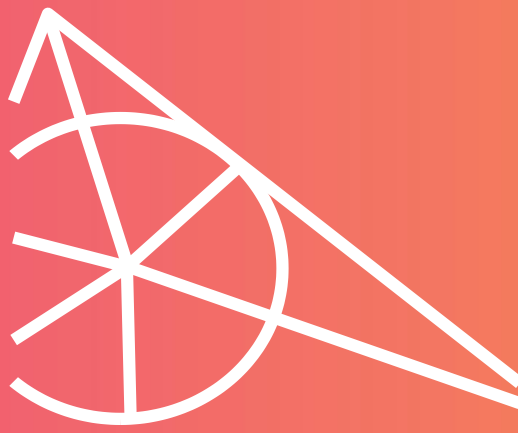
LESSON SCENARIO 06:
THE INCIRCLE OF A TRIANGLE



Topic: Geometry

Level: Age 14 -15

Foreknowledge: The circle inside a triangle, bisectors, the centre of a circle inside a triangle, the perimeter of a triangle, the area of a right triangle, heron's formula, the area of a disc.

Correlation: Woodwork, Art, Construction, Architecture

LEARNING OUTCOMES

- Build a circle inside a triangle
- Make the connection between the area, the perimeter and the radium of the circle inside the
- Apply the formula discovered in practical and concrete situations

TEACHING METHODS

- Practical work
- Hands-on activity
- Pair work

KEY WORDS

- Circle inside a triangle
- Triangle area
- Triangle semi perimeter
- Heron
- Radium

RESOURCES

- Board
- Geometrical material
- Handouts
- Scissors
- Videoprojector
- Laptop/ calculator

ACTIVITIES

ACTIVITY 1 (1 – 5 minutes)

The teacher refreshes the notions

The circle inside a triangle. The triangle's sides (considered segments) are tangents to the circle.

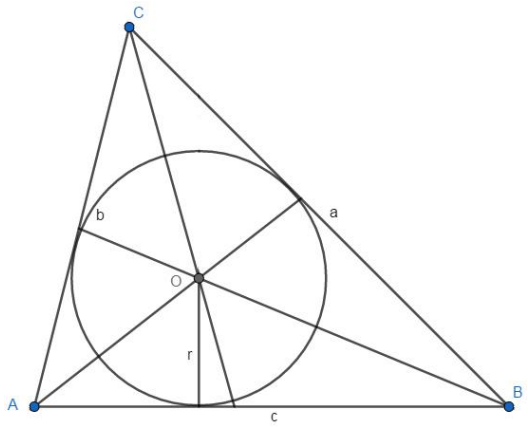
The centre of the circle inside a triangle lies at the intersection of its angles.

The bisector of a triangle is the semi right originating in the triangle's peak, which divides this angle in two other equal angles.

The semi perimeter of the triangle $p = \frac{a+b+c}{2}$.

The triangle area, using Heron's $S = \sqrt{p(p-a)(p-b)(p-c)}$.

The area of a disc $A = \pi r^2$.



ACTIVITY 2 (5 minutes)

The teacher enunciates the theorem:

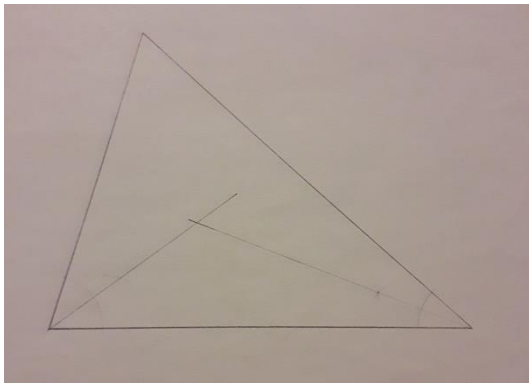
In any triangle, there is the inequality $r = \frac{S}{p}$ **, where** r **is the radium of the circle inside the triangle,** S **is the area of the triangle and** p **is the semi perimeter of the triangle,** $p = \frac{a+b+c}{2}$.

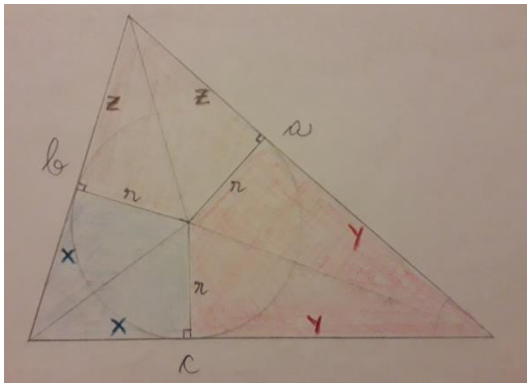
ACTIVITY 3 (10 minutes)

The theorem is demonstrated, the teacher makes the construction on the board and the students are working in pairs.

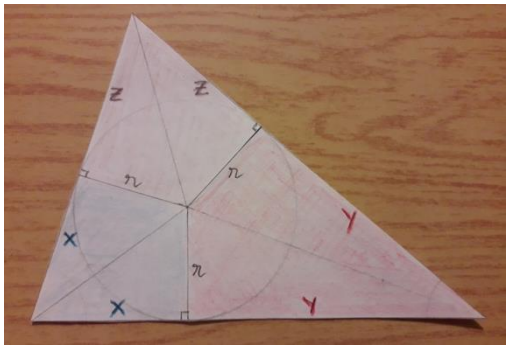
1. **take a piece of paper and draw a triangle. Inside the triangle draw a circle. From the centre of the circle, draw the tangential points.**

   $a + b + c = 2(x + y + z)$, **and you get** $p = x + y + z$

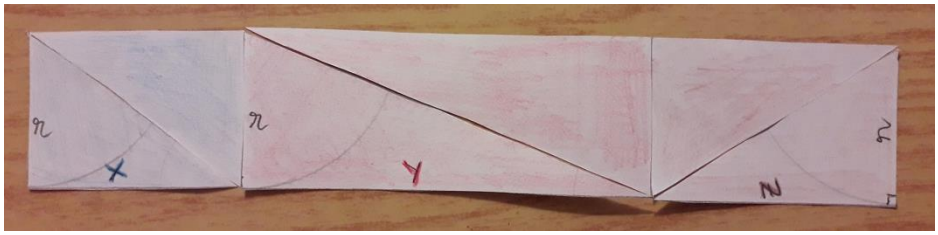




2. **Cut the triangle in the 6 newly resulted triangles.**



3. **Regroup the triangles in order to form a rectangle with a length side** r **and the other one of length** $x + y + z$



4. **The area of the initial triangle is equal to the one of the rectangles, so** $S = r(x + y + z)$, $S = rp$.

ACTIVITY 4 (15 minutes)

George the carpenter has to build a cupboard whose shelves have the shape of isosceles triangles, like in the drawing. Help him calculate the leg of the triangle knowing that the plates on the shelf will have a diameter of 40 cm.

a is one leg of the triangle.

Obviously, the other leg is also *a*. the hypotenuse is $a\sqrt{2}$.

In this way, the area of the triangle is $S = \frac{a^2}{2}$

and the perimeter $p = \frac{a+a+a\sqrt{2}}{2}$.

Substituting in the newly learned equality, we get:

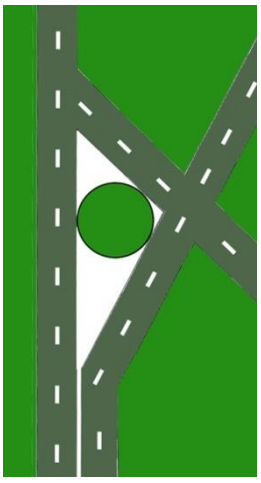
$$S = rp \Leftrightarrow \frac{a^2}{2} = \frac{a + a + a\sqrt{2}}{2} r \Leftrightarrow a = \left(2 + \sqrt{2}\right) r$$

for $r = 20$ şi $\sqrt{2} \simeq 1,42$ we get $a \simeq (2 + 1,42) \cdot 20 = 68,4$ cm

EVALUATION

Evaluation handout

For tasks 1 and 2 only the correct answer is required, but for 3 and 4 we require the whole calculation process.



(20p) 1. The centre of the circle inside a triangle is at the:

a) intersection of the *mediators*' sides of the triangle

b) intersection of the *medians* of the triangle

c) intersection of the angles' bisectors of the triangle

d) intersectio of the triangle's heights

(20p) 2. Heron's Formula for calculatingtheareaof a triangle is:

a) $S = \sqrt{p(p-a)(p-b)(p-c)}$, where S is the perimeter of the triangle

b) $S = \sqrt{p(p-a)(p-b)(p-c)}$, where S is the semiperimeter of the triangle

c) $S = \sqrt{(p-a)(p-b)(p-c)}$, where S is the perimeter of the triangle

d) $S = \sqrt{(p-a)(p-b)(p-c)}$, where S is the semiperimeter of the triangle

(20p) 3. Calculate the radium of a circle inside the triangle whose area is 96 m^{2}and the perimeter of 48 m.

(30p) 4. At the crossroads in the image the design company has to place a lawn in the circle in the middle and place white marble in the rest of the triangle. Help the workers calculate the lawn surface and the surface that has to be covered in marble, knowing the lengths of the triangle's sides: $a = 40$ m, $b = 30$ m şi$c = 20$ m.

Teacher'spoints: 10

Working time: 15 minutes

key:

1. c)

2. b)

3. We calculate the semi perimeter of the triangle $p = \frac{a+b+c}{2} = 24$m.

We calculate the radium of the circle inside the triangle $r = \frac{S}{p} = 4$m.

4. We calculate the semi perimeter of the triangle $p = \frac{a+b+c}{2} = 45$m.

We calculate the surface of the triangle using Heron's formula $S = \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{45 \cdot 5 \cdot 15 \cdot 25} = 75\sqrt{15}$m².

We calculate the radium of the circle inside a triangle $r = \frac{S}{p} = \frac{75\sqrt{15}}{45} = \frac{5\sqrt{15}}{3}$m.

We calculate the disc's surface $A = \pi r^2 = \pi\left(\frac{5\sqrt{15}}{3}\right)^2 = \pi\frac{125}{3}$ m².

The surface that has to be covered in marble is $S - A = 75\sqrt{15} - \pi\frac{125}{3}$ m².

INCLUSIVENESS GUIDENESS

Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- **When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.**
- **You can use checklists for your students to make sure they have done all the steps**
- **Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.**
- **At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process**
- **Make sure that the material the students manipulate is easy enough to grasp**
- **While using different media (paper, computer and visual aids) choose different background than white which can be too bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colours to learn more about student's preference.**
- **To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.**

EXAMPLE:

1. Start every lesson with a short "CHECK-IN"

- **Today, we will study the topic (name of the topic)**
- **I will tell you about: (name 3 keywords connected with the topic)**
- **Then I will present exercises: (name the exercises from the student book)**
- **Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)**
- **Once the exercises will be done [To continue]**

2. Then finish lesson with a short "CHECK-OUT"

- **During the lesson we learn about (topic of the lesson)**
- **The most important things were: (name 3 keywords connected with the topic)**
- **We were able to do… (tell about the work student done during the lesson)**
- **We will explore the topic next time when we will learn about (name the following topic)**

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.