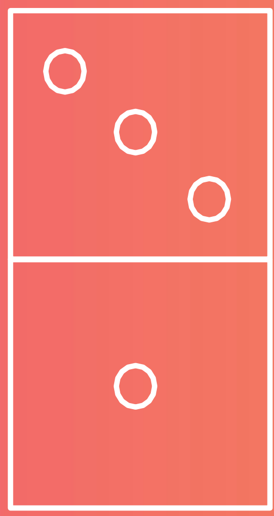
LESSON SCENARIO 11:
ROLL THE DICE



Topic: Conditional probability

Level: Age 14 -16

Foreknowledge: Basics of statistics

Correlation: Psyhology, Games

LEARNING OUTCOMES

- Find conditional probability
- Realize false positives
- Collect data
- Visualise conditional probability

TEACHING METHODS

- A practical experiment which uses tree diagrams to help students understand the nature of questions in conditional probability.
- Problem solving approach

KEY WORDS

- Probability
- Data analysis
- Independent and dependent events
- Tree diagram

RESOURCES

- A simple dice for every group
- About 7 red, 22 green, 19 blue and 11 yellow cubes or coloured pieces of paper per group
- A worksheet

ACTIVITIES

STEP 1: Describe and simplify the problem. Provide a plan on how to tackle it.

INTRODUCTION TO THE LESSON (5 minutes)

Describe and simplify the problem. Provide a plan on how to tackle it

What is probability? Probability is a measure quantifying the likelihood that events will occur.

An understanding of probability is important in medicine, law and wider society. Today we are going to work on a problem that models the interpretation of statistics for testing eg. getting test result for serious diseases like cancer, checking athletes for using illegal substances and many other similar situations involving false positives.

So, probability can be very serious but it can also provide answers to everyday problems. Imagine:

A food manufacturer has a big factory where different types of food are produced. Every production line is separated so ketchup and nut chocolate are produced in separate buildings. As we know, nut allergies can be dangerous for many people. The company is very cautious, so they put information on ketchup labels that „it may contain nuts"but there is an inspection coming up, during which the inspector is very suspicious and he picks 24 bottles for testing.

But what are the odds that the nuts are actually going to be in the ketchup bottle?

We suppose that a part of the bottles will contain nuts and a part of them will be nut free, but some of them will represent a „false positive", so what is the chance that the producer will be wrongly accused of not being careful?

STEP 2: Carry out an investigation and collect data

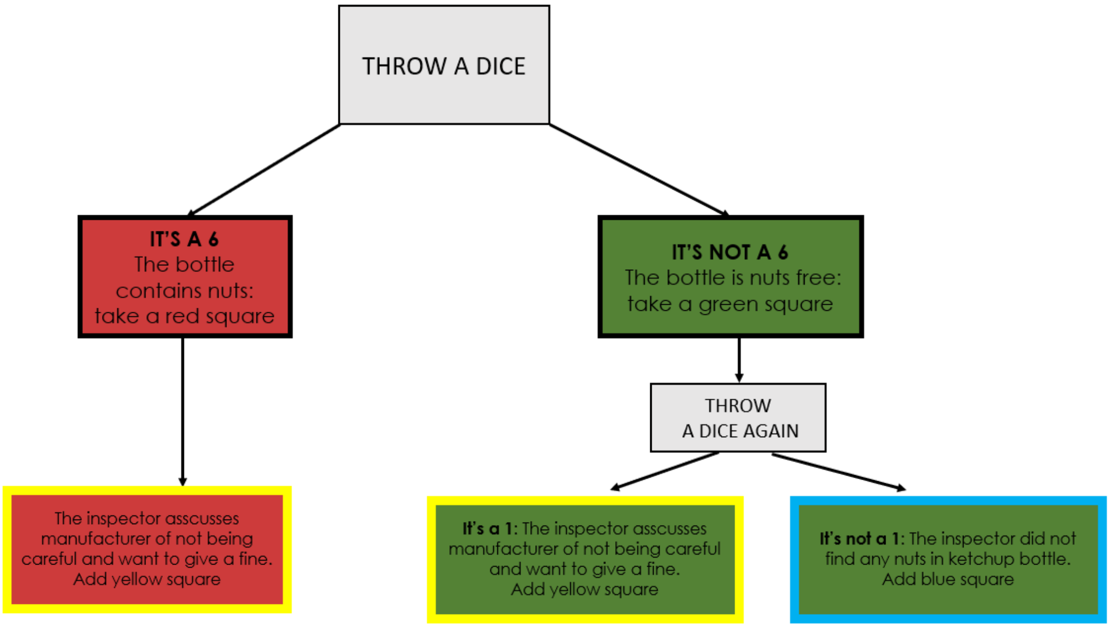
5 minutes: Divide students into groups of 3 or 4. Each group will need one 6-sided dice, and some red, green, blue and yellow squares.

Throw the dice! 6 = The bottle contains nuts, anything else = the bottle does not contain nuts.

If the bottle contains nuts, there is no need to throw the dice again.

If the bottle is nut free, throw the dice again: 1 = It is a false positive, anything else= it is negative.

First, show a trial test, so students will know how to run their own investigation.



Ask the students: Can you tell me what these colours mean?

[red=positive; green=negative; yellow=suspicious, getting a fine (positive or false positive); blue=negative and no fine]

TASK (10 minutes)

Ask the students to repeat the experiment 24 times (as there are 24 bottles). Each group should end up with 24 squares.

TASK (5 minutes)

Ask the students to fill the data in their worksheets. One worksheet for 1 group.

TASK (15 minutes)

After this experiment, compare the outcomes of the groups' experiments with the actual probabilities and make sure they understand why there is a difference. For this, review how probabilities are calculated:

The positive bottles: $\frac{1}{6} \cdot 24 = 4$

$$\frac{1\ (\text{only one number } (6) \text{ indicates a positive})}{6\ (\text{six options on the dice})} \cdot 24\ (\text{total number of bottles}) = 4$$

The negative bottles: $\frac{5}{6} \cdot 24 = 20$

$$\frac{5\ (\text{five numbers } (1,2,3,4,5) \text{ indicate a negative})}{6\ (\text{six options on the dice})} \cdot 24\ (\text{total number of bottles}) = 20$$

The false negative bottles: $\frac{1}{6} \cdot 20 = 3.\dot{3}$

$$\frac{1\ (\text{only one number } (1) \text{ indicates a positive})}{6\ (\text{six options on the dice})} \cdot 20\ (\text{total number of negative bottles}) = 3{,}\dot{3}$$

The actual negative bottles: $\frac{5}{6} \cdot 20 = 16.\dot{6}$

$$\frac{5\ (\text{five numbers } (2,3,4,5,6) \text{ indicate a negative})}{6\ (\text{six options on the dice})} \cdot 20\ (\text{total number of negative bottles}) = 16{,}\dot{6}$$

To go a little further, you can also introduce the following false positive rate formula to check if the calculated results are correct and to make sure that your students understand the notion of false positives:

$$\frac{FP}{N} = \frac{FP}{FP + TN}$$

FP = number of false positives

TN = number of true negatives

N = total number of negatives

FP False positives = Number of yellow cards/cubes – number of red cards/cubes

TN True negatives = Number of blue cards/cubes

N Negatives = Number of green cards/cubes

TASK (10 minutes)

Discuss with your students:

- What were your expectations before starting the experiment? Compare your results with your classmates. Who was nearest to the actual probabilities we just calculated?
- What is the probability for the manufacturer to get a fine for a bottle that was actually nut free?
- What are the advantages of the tree diagram? How about the 2-way table? What information can you find easily from each, what is more difficult or impossible to find? What part of this exercise did you find difficult?

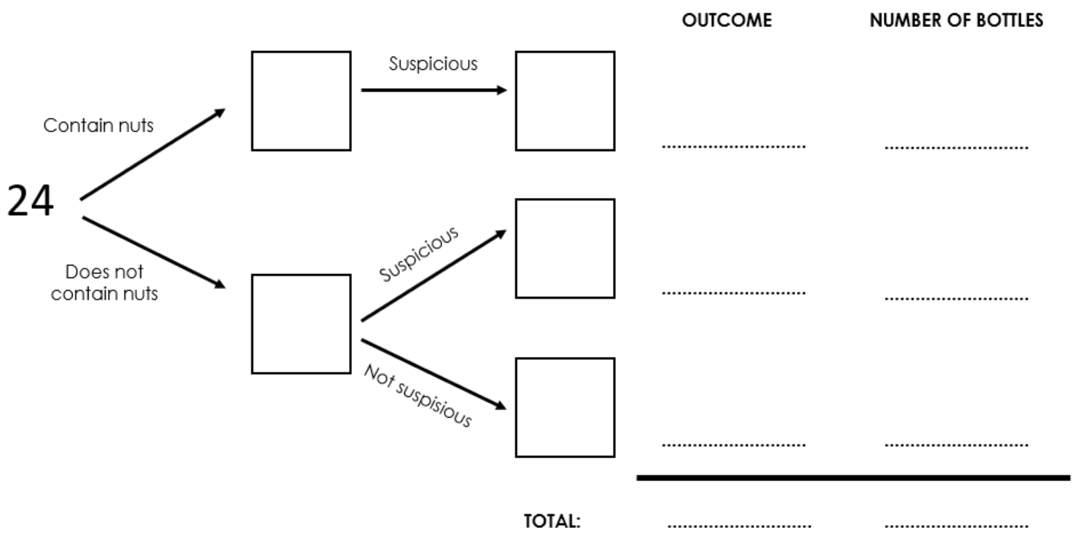
WORKSHEET

COMPLETE THE 2-WAY TABLE AND TREE DIAGRAM BELOW FOR YOUR TEST RESULTS FOR 24 BOTTLES OF KETCHUP.

2 WAY TABLE

2 way table		Does the inspector find the bottle suspicious and want to give a fine?		
		Very suspicious of the bottle – want to give fine	Did not find the bottle suspicious – no fine	TOTAL
Is the ketchup bottle nut free?	Nut free			
	Contains nuts			
	TOTAL			

TREE DIAGRAM



Answers:

The answers below are the different outcomes from the experiment. The first and third table shows the calculated results whereas the second table shows one possible outcome when conducting the experiment with a dice following the instructions above. The numbers will of course always vary as 24 is a too small sample size to achieve precise results.

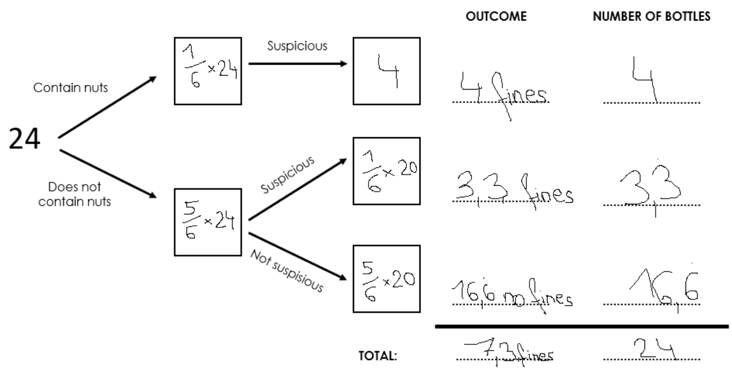
2 WAY TABLE: Calculated results

2 way table		Does the inspector find the bottle suspicious and want to give a fine?		
		Very suspicious of the bottle – want to give fine	Did not find the bottle suspicious – no fine	TOTAL
It the ketchup bottle nut free?	Nut free	3,3	16,6	20
	Contains nuts	4		4
	TOTAL	7,3	16,6	24

2 WAY TABLE: possible results from experiment

2 way table		Does the inspector find the bottle suspicious and want to give a fine?		
		Very suspicious of the bottle – want to give fine	Did not find the bottle suspicious – no fine	TOTAL
It the ketchup bottle nut free?	Nut free	5	13	18
	Contains nuts	6		6
	TOTAL	11	13	24

TREE DIAGRAM: Calculated results



EVALUATION

Explain what “false positive” means!

Why were the results from the experiment different to the calculated results?

What would be different regarding the probabilities if the dice had 8 sides?

1. **Explain what “false positive” means!**

False positive means that a positive result is given when the true value is negative.

In our case it means that some of the suspicious bottles (number of “yellow” – number of “red”) do not actually contain nuts, but as the second round of testing (second throw of the dice) indicates that they do, the manufacturer will get a fine for them, even though there are no nuts in them.

2. **Why were the results from the experiment different to the calculated results?**

24 is a too small sample size to achieve precise results.

3. **What would be different regarding the probabilities if the dice had 8 sides?**

There would be less false positives as the probability would go down to 1/8 from 1/6.

- Visual support is beneficial for all the students: the teacher can draw a diagram with the dice on the board so the student can look while they conduct the experiment. Also encourage students to illustrate problems if it helps them, e.g. mind-mapping.
- Split the instructions into successive steps. It is better to present instructions both in a written and oral form.
- Make sure that the students understand the concepts, ask students to repeat the instructions. Make sure students feel comfortable asking questions.
- Authorize supports and assistive technology for students who need them.
- Make sure the font, line spacing and alignment is accessible for students with learning disorders.
- Ask the students to interpret the problem to develop their own analytical skills.
- Ask for constant feedback in order to adapt the exercises. Some students might need more time than others. Also provide required materials ahead so time is not lost searching and gathering them.
- Provide seating away from noises and distraction for students with learning disabilities, especially TDA/TDAH.
- Encourage students to create a math dictionary with concepts explained in their own way with words, pictures, primary language, etc.
- Help students understand the connection of this exercise with real life problems and the value of probabilities in everyday life, making it more tangible.
- Bear in mind that some students might have difficulties explaining the process they have gone through to find the answers, establish a safe and inclusive environment for these students to reduce anxiety. Each student should be able to find their mental strategy to solve problems.

EXAMPLE:

1. Start every lesson with a short “CHECK-IN”
   - Today, we will study the topic (name of the topic)
   - I will tell you about: (name 3 keywords connected with the topic)
   - Then I will present exercises: (name the exercises form the student book)
   - Then we will do exercises (explain the way the student will be working: ex. together with teacher / in pairs / individually)
   - Once the exercises will be done [To continue]

2. Then finish lesson with a short “CHECK-OUT”

- **During the lesson we learned about (topic of the lesson)**
- **The most important things were: (name 3 keywords connected with the topic)**
- **We were able to do… (talk about the work students have done during the lesson)**
- **We will explore the topic next time when we will learn about (name the following topic)**

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.