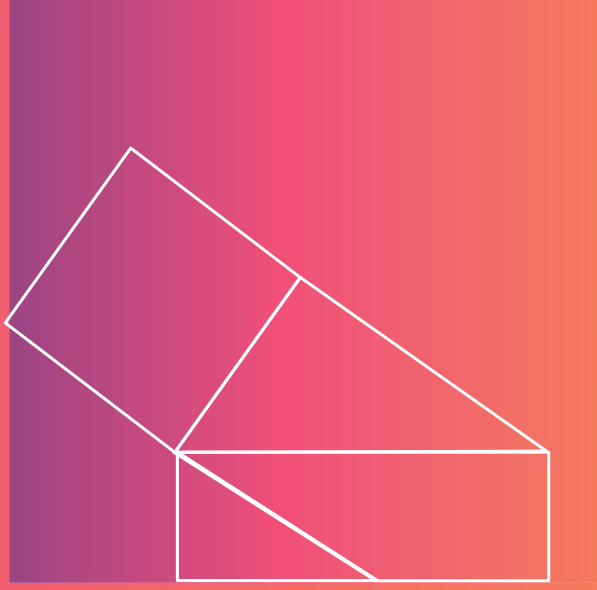
LESSON SCENARIO 12:
RIGHT TRIANGLE THEOREM



Topic: Geometry

Level: Age 15 -16

Foreknowledge: Parts of a right-angled triangle (legs, altitude, projections, etc.), Pythagorean Theorem, Proportions

Correlation: none

LEARNING OUTCOMES

- Learn the two Euclid's theorems
- Apply the theorems in an algebraic form
- Apply the theorems in a proportional form

TEACHING METHODS

- Practical work
- Hands-on activity
- Group work
- Brainstorming

KEY WORDS

- Right angle
- Triangle
- Leg
- Projection
- Altitude
- Hypothenuse

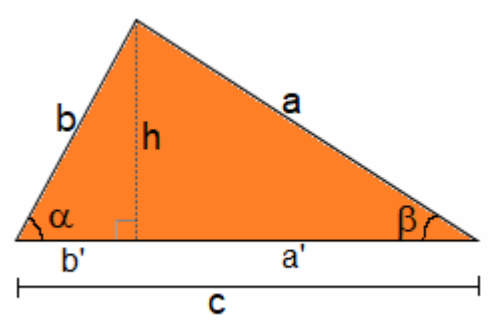
RESOURCES

- Triangle rulers
- Coloured cardstock
- Scissor

ACTIVITIES

INTRODUCTION TO THE LESSON (5 MIN)

The teacher reminds the students the various parts of a right-angled triangle. In particular the definitions of legs, projections of legs, hypotenuse, altitude, as in the figure:



a and b: major and minor leg respectively

h: altitude to the hypotenuse

c: hypotenuse

a': projection of a on c

b': projection of b on c

α: angle opposite to a (and thus a is the side opposite to α)

β: angle opposite to b (and thus b is the side opposite to β)

The teacher asks the students to note this nomenclature, since it will be used during the hands-on part of the lesson.

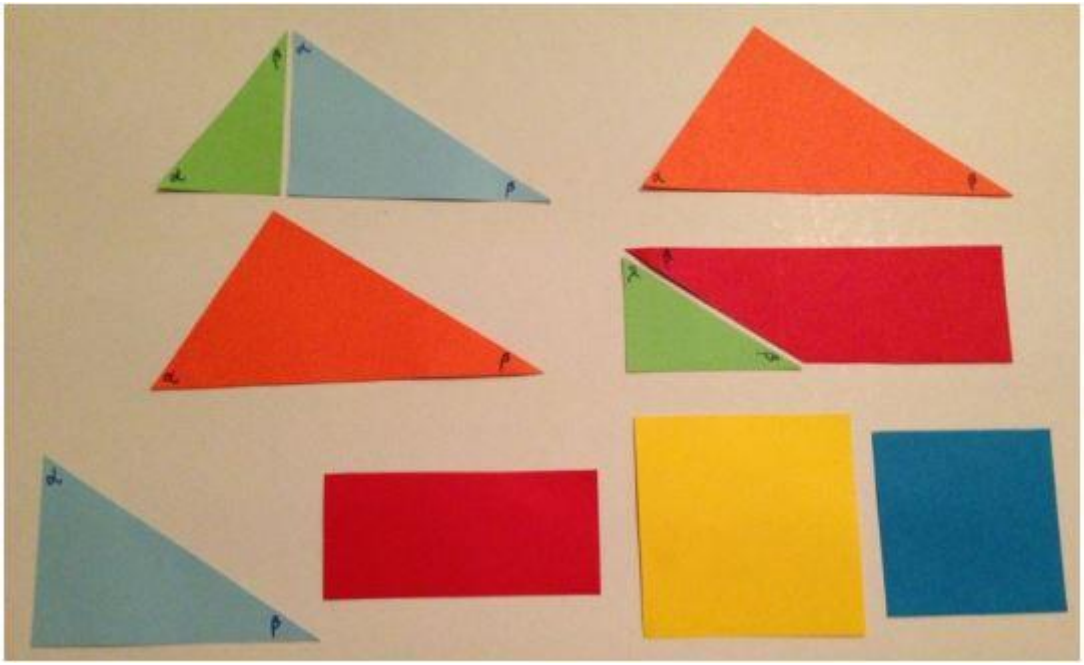
Then the teacher divides the class in groups, depending on the total number of students (optimum is 4/5 students per group), and explains to the students that they are going to prove two important rules concerning right-angled triangles: Leg Rule and Altitude Rule.

HANDS-ON PART 1 (10 - 15 minutes)

The teacher asks each group to prepare the following cardstock figures:

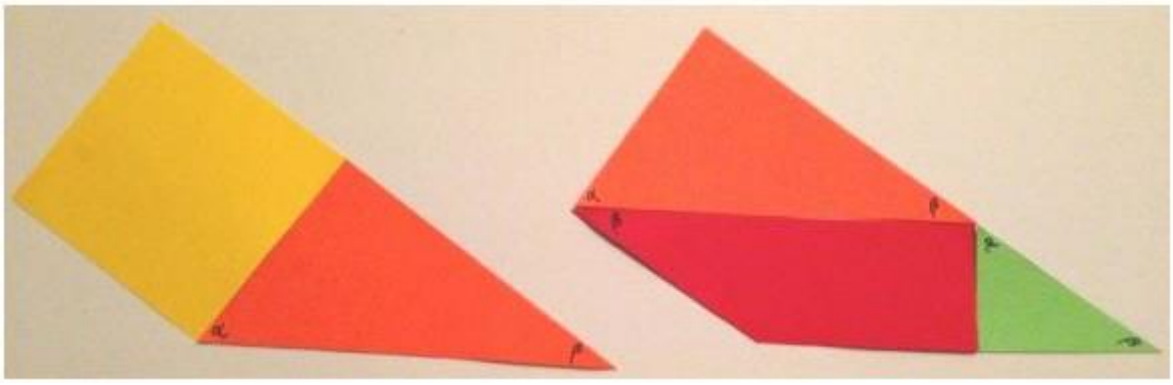
- **Two orange right-angled triangles, with angles and sides named as the previous figure;**
- **a yellow square with side b**
- **a blue square with side h**
- **a red rectangle with sides a' and b'**
- **two green right-angled triangles with legs b' and h**
- **two light blue right-angled triangles with legs a' and h**
- **a rectangle trapezoid with major base c, minor base (c - h) and altitude b'**

See figure below:



HANDS-ON PART 2 (15 - 20 minutes)

The students have now to build the following equivalent figures:



By overlapping these two figures, and noticing that they have exactly the same shape, the students will obtain:

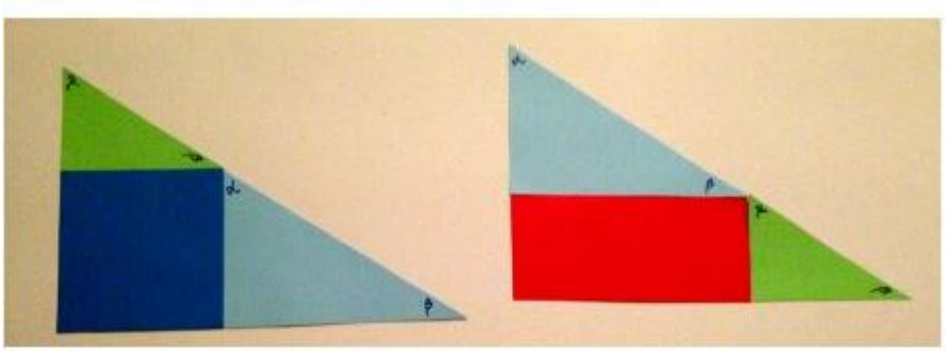
$$b^2 = c \cdot b'$$

that represents the leg rule (known in Italy as First Euclid's Theorem)

(it can be useful to note that the red trapezoid plus the green triangle is equivalent to a rectangle, as in the figure on the right):

The teacher helps the students to state this rule. They should come to the following statement: "The square built on a leg is equivalent to the rectangle whose sides are the hypotenuse and the projection of the leg on the hypotenuse".

In the same way, building the following equivalent figures:



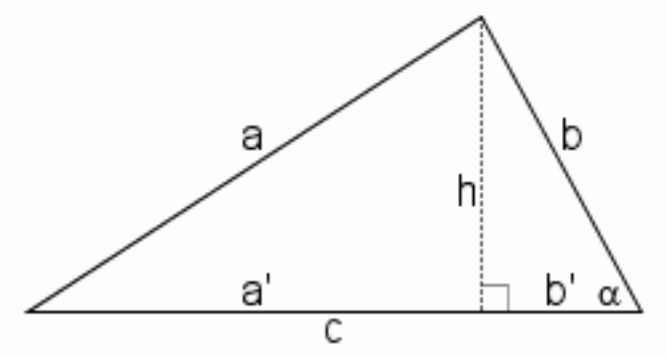
and by overlapping them, the students will obtain:

$$h^2 = a' \cdot b'$$

that represents the altitude rule (known in Italy as Second Euclid's Theorem)

The teacher helps the students to state this rule. They should come to the following statement: "The square built on the height corresponding to the hypotenuse is equal to the rectangle whose sides are the projections of the two legs on the hypotenuse".

LAST TASK (10 - 15 minutes)



The teacher writes on the blackboard the statements of First and Second Euclid's Theorem in the proportional form:

1) The leg of a right triangle is the mean proportional between the hypotenuse and the projection of the leg on the hypotenuse

$$c : a = a : a' \qquad \text{for leg a}$$

$$c : b = b : b' \qquad \text{for leg b}$$

2) The altitude to the hypotenuse of a right triangle is the mean proportional between the segments into which it divides the hypotenuse, i.e. the two projections (see picture below)

$$a' : h = h : b'$$

The teacher now asks the students to verify the equivalence of the two forms (the algebraic one, deducted from the figures, and the proportional one).

A discussion follows (brainstorming phase).

EVALUATION

1. DO I KNOW THE EUCLIDE'S THEOREMS?

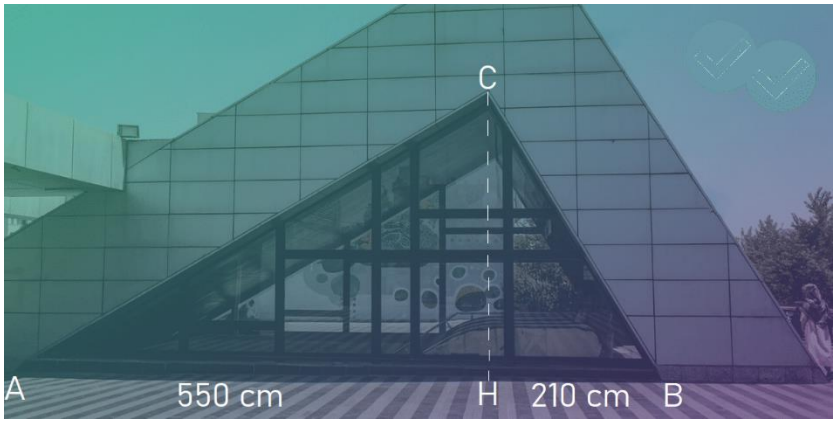
2. CAN I APPLY THEM?

3. CAN I COMPARE DIFFERENT WAYS OF STATING THE THEOREMS?

The students must answer these questions in 5 minutes:

A famous architect, keen on geometry, designed this weird building.

Using a tape measure, you find out the length of the base, from A to H, and from H to B (see figure).



1) What is the maximum inner height CH of the building? Which theorem is needed to answer?

2) Suppose you want to put Christmas lights all along the glass facade (side AC and CB). How many centimetres of decorating string are necessary? Which theorem is needed to answer?

INCLUSIVENESS GUIDELINES

Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.
- You can use checklists for your students to make sure they have done all the steps
- Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.
- At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process
- Make sure that the material the students manipulate is easy enough to grasp
- While using different media (paper, computer and visual aids) choose different background than white which can be too bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colours to learn more about student's preference.
- To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.

EXAMPLE:

1. Start every lesson with a short "CHECK-IN"

- **Today, we will study the topic (name of the topic)**
- **I will tell you about: (name 3 keywords connected with the topic)**
- **Then I will present exercises: (name the exercises form the student book)**
- **Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)**
- **Once the exercises will be done [To continue]**

2. Then finish lesson with a short "CHECK-OUT"

- **During the lesson we learn about (topic of the lesson)**
- **The most important things were: (name 3 keywords connected with the topic)**
- **We were able to do… (tell about the work student done during the lesson)**
- **We will explore the topic next time when we will learn about (name the following topic)**

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.