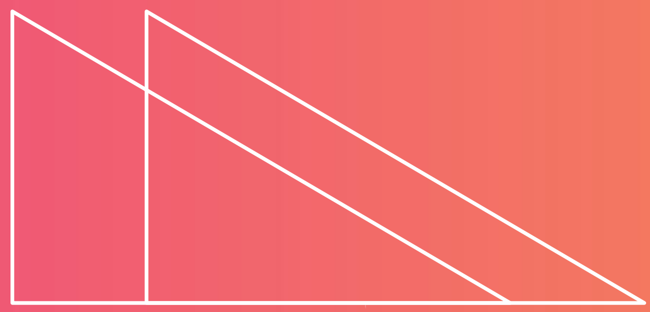
LESSON SCENARIO 13: THALES’S THEOREM



Topic: Geometry

Level: Age 14 -15

Foreknowledge: Elementary mathematic operations, solving linear equations with one unknown

Correlation: Everyday life, Geometry

LEARNING OUTCOMES

- Students will learn about Thales's Theorem
- They will be able to apply one of the criteria of similar triangles based on one problem that had been drawn from the History of Mathematics

TEACHING METHODS

- Hands-on activity
- Group work

KEY WORDS

- Thales's theorem
- Similar triangles

RESOURCES

- Whiteboard
- Worksheets

ACTIVITIES

ACTIVITY 1 (15 minutes)

The teacher introduces Thales of Miletus and his theorem to the students. The teacher can advise the students to read on their own who was Thales of Miletus from their handouts.

Who was Thales of Miletus?

Thales of Miletus was born in 624BC in Miletus, Greece. He is considered as the premier pre-Socratic philosopher, the first of the seven sages of antiquity. He was mathematician, physicist, astronomer, engineer, meteorologist. He was the founder of the Ionian School of natural philosophy in Miletus.

Aristotle and other ancient philosophers considered Thales as the first Greek philosopher; Thales was the one who achieved to approach and explain the natural phenomena through scientific logic, refusing to accept any previous interpretations of natural phenomena, which until then had been based solely on myths, legends and religious beliefs. Hence, Thales of Miletus has been considered as the one who first paved the way for scientific research.

The teacher will then proceed by stating the theorem of Thales and explaining it to the students:

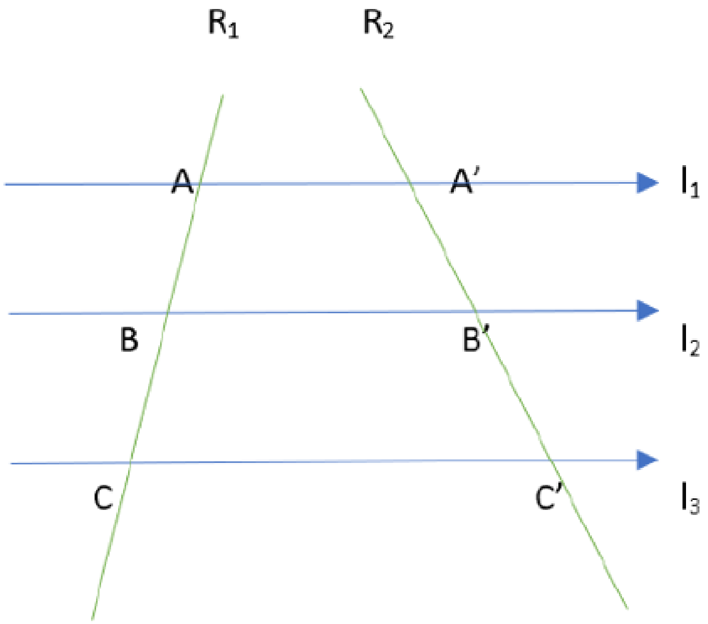
Thales' theorem

Thales of Miletus is being widely known for his theorems in the field of geometry. One of them, is the theorem presented below:

If we have three parallel straight lines L_1, L_2 and L_3 which they cut (intersect) other two ones, namely R_1 and R_2, then they produced proportional segments.

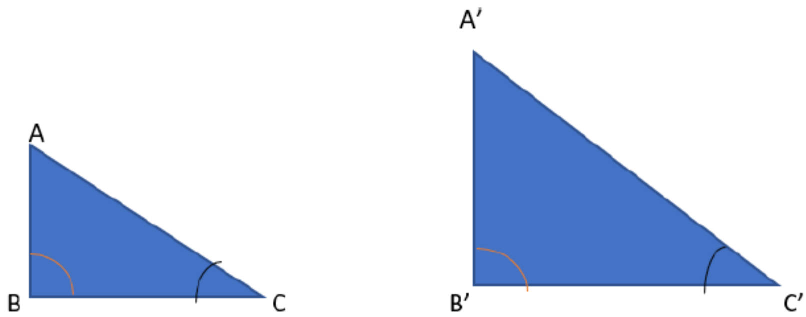
That is, if $L_1 // L_2 // L_3$ and they intersect R_1 and R_2, then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$

The teacher can further explain the above statement and its correlation with similar triangles using the example on the handout:



Moreover, the theory of similar triangles is strongly correlated with Thales theorem. Specifically, there are three criteria of similarity; here we will focus on the second criterion of similarity (usually found as AA).

If two triangles have two of their angles equal (one by one) then they are similar triangles



Criterion of similarity, which is being formed as follows:

Let's suppose that angle B of triangle ABC is equal to angle B' of A'B'C' and that angle C is equal to the angle C'. Then, according to the AA criterion of similarity given above, we can conclude that the triangles ABC and A'B'C' are similar, thus getting the following

$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = \lambda$, where λ is called "similarity ratio"

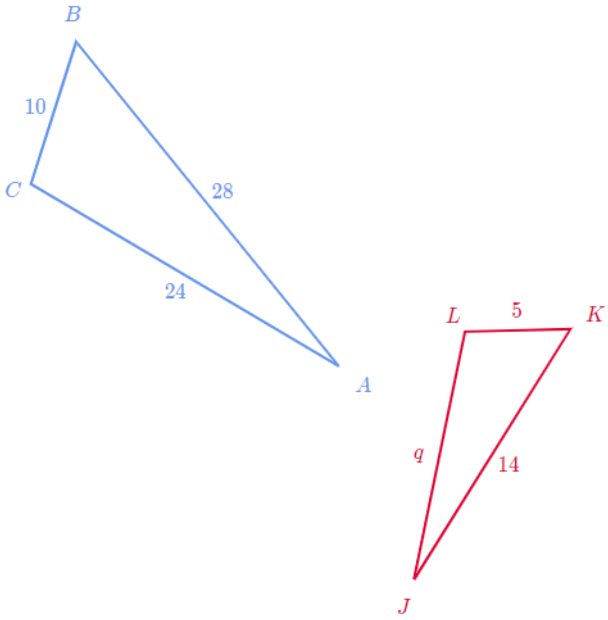
proportion:

ACTIVITY 2 (15 minutes): Students can use the theorem in practice

It is being suggested that the teacher asks the students to solve a simpler exercise on similar triangles before proceeding to the task. The teacher can use the following example on similar triangles:

The teacher can draw the below triangles on the whiteboard.

Triangle ABC is similar to JKL. Solve for q.



(Source: https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-solving-similar-triangles/e/solving_similar_triangles_1)

Solution: Before asking the students what the solution is the teacher could say that in this case, we can use the mentioned theorem $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$. This is because the exercise mentions at the beginning that the triangles are similar which is strongly correlated with Thales's theorem, as mentioned above.

For each step of the answer, the teacher can ask a different student so that more students participate.

Step 1: which sides of triangles ABC and JKL are similar?

The solution is: $\frac{AB}{JK} = \frac{BC}{LK} = \frac{CA}{JL}$ **.**

Step 2: What is the next step? To substitute the dimensions - gives us: $\frac{28}{14} = \frac{10}{5} = \frac{24}{q}$

Therefore, the "similarity ratio" of triangles ABC and JKL is equal to two.

q = 12.

<u>INTRODUCING THE EXERCISE AND ACCOMPLISHMENT OF THE TASK (40 min):</u>

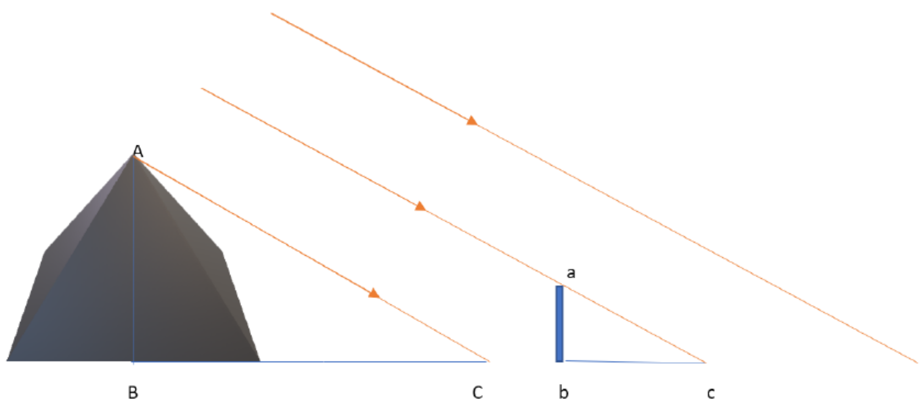
Advice: the teacher could solve the following questions with the students in the form of a discussion (with the whole classroom). Could give 5 – 10 minutes for each question before the correct answer is discussed.

TASK

Based on the history of mathematics, and according to Plutarch (Ancient Greek Essayst), Thales of Miletus used the theory of congruent triangles in order to solve a practical problem which had arisen at his ages. It is said that since then, nobody had managed to calculate the height of the pyramid of Cheops, due to the peculiarities of its shape (it had been built sideways).

However, Thales managed to solve this problem by calculating the length of the pyramid's shadow, thus earning the admiration of the Egyptian King, Amasis.

The following picture depicts Thales's solution:



Sideways of the pyramid, Thales placed a stick in parallel with the pyramid, whilst he immediately observed the shadow of the stick on the ground. Subsequently, he realized that the length of the stick (ab), the length of the stick's shadow (bc), as well as the length of the pyramid's shadow (BC) were all easily measurable quantities. Accordingly, he managed to count the pyramid's height by applying the first criterion of congruence in the two triangles that had been shaped.

Observe the picture above and work on the following questions:

Question 1: Which two triangles did he use in order to apply the AA criterion of similarity? Use the letters given in the picture above to define the triangles.

Answer1: The triangles are: triangle ABC and triangle abc

Question 2: How did Thales of Miletus prove that he could apply the specific criterion of similarity? In other words, how did he know that the prerequisites stated in the AA criterion of similarity were valid for this specific case?

Answer 2: The prerequisites of the AA criterion of the similarity are the following:

The two triangles should have two of their angles equal, one by one.

- In this case, the angle B is equal to the angle b inasmuch both the segments AB and ab are perpendicular to the ground, thus forming a right angle in both cases.
- Concurrently, the angle C is equal to the angle c. "Thales applied the experiment at a particular time of the day during which the rays of the sun were sideways of the pyramid" is being stated within the instructions of the tasks. This implies that the rays of the sun were parallel at this time, which means that the angle C is equal to the angle c.

Accordingly, we have proved that the two triangles have two of their angles equal, one by one, a fact that signifies that Thales was allowed to use the specific criterion.

Question 3: Which is the proportion that Thales formed in order to estimate the height of Cheops's pyramid?

Answer 3: $\frac{AB}{ab}=\frac{BC}{bc}$ where AB is the height of the pyramid

Question 4: Let's suppose that the length of the stick was 2 feet, the length of its shadow was 4 feet, while the length of the pyramid's shadow was 912 feet. By applying the proportion of 'Question 3', calculate the height of the Cheops's pyramid.

Answer 4: AB is the height of the pyramid

ab = 2

bc = 4

BC = 912

$\frac{AB}{2} = \frac{912}{4}$

$\frac{AB}{2} = 228$

AB = 228 x 2

AB = 456 feet

Question 5: Calculate the similarity ratio.

Answer 5:

$\lambda = \frac{AB}{\alpha\beta} = \frac{BC}{bc}$

$\lambda = \frac{912}{4} = \frac{456}{2} = 228$

EVALUATION

1. DO I KNOW Thales'sTheorem?
State the theorem.

2. DO I UNDERSTAND how to applyThales'sTheorem when using similar triangles?

3. CAN I EXPLAIN IT to my classmates?

INCLUSIVENESS GUIDELINES

Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- **When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.**
- **You can use checklists for your students to make sure they have done all the steps**
- **Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.**

- At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process
- Make sure that the material the students manipulate is easy enough to grasp
- While using different media (paper, computer and visual aids) choose different background than white which can be to bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colors to learn more about student's preference.
- To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.

EXAMPLE:

1. Start every lesson with a short "CHECK-IN"
   - Today, we will study the topic (name of the topic)
   - I will tell you about: (name 3 keywords connected with the topic)
   - Then I will present exercises: (name the exercises form the student book)
   - Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)
   - Once the exercises will be done [To continue]

2. Then finish lesson with a short "CHECK-OUT"
   - During the lesson we learn about (topic of the lesson)
   - The most important things were: (name 3 keywords connected with the topic)
   - We were able to do… (tell about the work student done during the lesson)
   - We will explore the topic next time when we will learn about (name the following topic)

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.