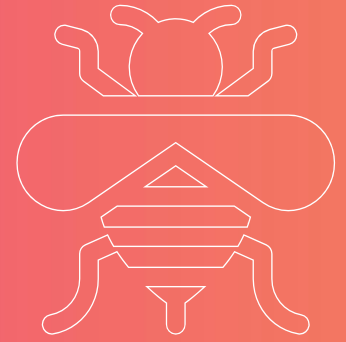


# LESSON SCENARIO 14: SYSTEM OF LINEAR EQUATIONS



Topic: Linear Equations

Level: Age 13 -15

Foreknowledge: Elementary mathematic operations, solving linear equations with one unknown

Correlation: Non-formal games, brain teasers

## LEARNING OUTCOMES

- Students will practice, through a gamified method, on how to find the solution for a linear system of equations, utilizing the method of substitution

## TEACHING METHODS

- Hands-on activity
- Group work

## KEY WORDS

- System of Linear Equations
- Set of Variables
- Solution of a System

## RESOURCES

- Pieces of paper

## ACTIVITIES

### INTRODUCTION TO LINEAR EQUATIONS (20 MIN)

The teacher begins by explaining to the students the definition of a system of linear equations saying that is being composed of two or more linear equations which employ the same set of variables". The teacher can give a motivational example in the form of a short story. For example: a boy and a girl visit a pet shop. The boy buys 1 gold fish and 1 clown fish at the price of 10 euro and the girl buys 2 gold fish and 3 clown fish at the price of 25 euro. Let  $x$  represent the gold fish and  $y$  the clown fish. The teacher can help the students find the two linear equations and explain to them the method of substitution to find the solutions of  $x$  and  $y$ .

Slightly after, he makes an introduction to the method of substitution, as a method of solving a linear system with two equations and two variables. All the theory that must be taught is given below:

In mathematics, a system of linear equations is being composed of two or more linear equations which employ the same set of variables. Solution of a linear system is one assignment of values which satisfy all the equations of the system at the same time. For instance, for the following linear system of two equations in two variables  $x, y$ :

$$x + 2y = 7$$

$$x - y = 1$$

The solution is given by the assignment  $x = 3$  and  $y = 2$ , in as much such assignment of values makes both equations valid at the same time.

The same applies for a linear system of three equations in three variables  $x, y, z$ , such as the following:

$$x + 2y + z = 9$$

$$x - y - 2z = -3$$

$$x + y + z = 6$$

in which  $x = 2, y = 3$  and  $z = 1$  or  $(x, y, z) = (2, 3, 1)$  is the solution of this linear system.

Within the current tool, we will focus on the method of substitution, as a method of resolving a linear system of equations. We will try to explain the method through the following example, a linear system which involves 2 equations in 2 variables:

$$2x + 3y = 8$$

$$4x - 5y = -6$$

As a first step, we solve one of the two equations for  $x$  in terms of  $y$ , or for  $y$  in terms of  $x$ . In this case we choose to solve the first equation for  $x$  in terms of  $y$ :

$$2x + 3y = 8$$

$$2x = 8 - 3y$$

$$x = \frac{8-3y}{2}$$

$$x = 4 - \frac{3y}{2}$$

At this stage, we substitute this expression of  $x$  into the other (second) equation of the linear system. Hence the equation  $4x - 5y = -6$  will take the form of:  $4(4 - \frac{3y}{2}) - 5y = -6$

$$16 - 6y - 5y = -6$$

$$16 - 11y = -6$$

$$-11y = -22$$

$$\frac{-11y}{-11} = \frac{-22}{-11}$$

$$y = 2$$

Now, we substitute  $y = 2$  back into any equation of the system that involves the variable  $x$ . For instance, we substitute  $y = 2$  into the equation  $2x + 3y = 8$ :

$$2x + 3 \cdot 2 = 8$$

$$2x + 6 = 8$$

$$2x = 2$$

$x = 1$ . So, the solution is  $x = 1$  and  $y = 2$  or  $(x, y) = (1, 2)$

### STUDENTS ARE PRACTISING ON THE METHOD OF SUBSTITUTION (10 min)

It's being suggested that the teacher will ask students to solve on their own at least 2-3 linear systems of two equations so as to get familiarised with the taught methodology before proceeding to the TASK which is embedded in the worksheet. Accordingly, this tool can be used as a validation that the students have indeed fully understood the way of solving such systems, and by no means as a tool for the introduction of the theory on a linear system with two equations and two variables.

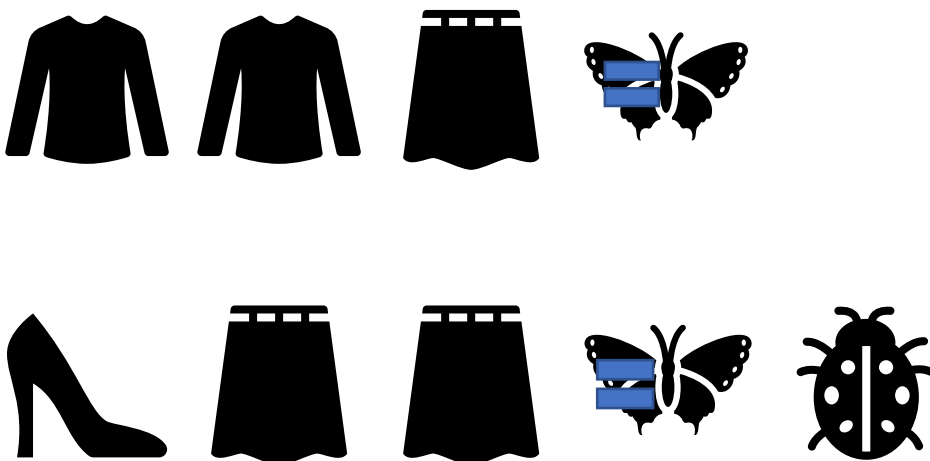
### INTRODUCTION OF THE EXERCISE AND ACCOMPLISHMENT OF THE TASK (20 MIN)

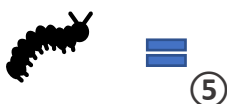
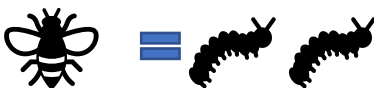
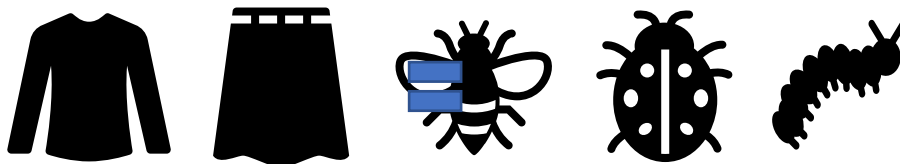
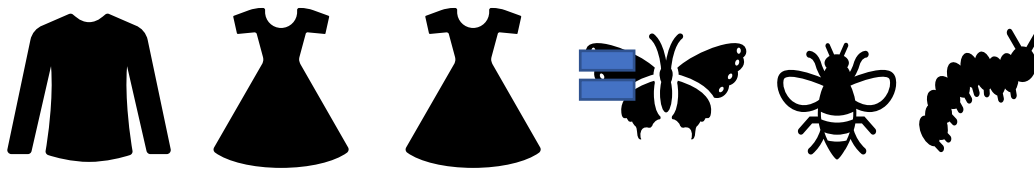
The teacher asks the students to form pairs in order to work together and solve the following "brain teacher", using the method of substitution.


### WORKSHEET FOR STUDENTS:

#### TASK

Using the method of substitution presented above, try to find an arithmetic value for all the items contained within the following system:





After the introduction of the task, the teacher asks the students if they have any ideas on how to solve the task. One guiding question could be: “How do we start?”. The answer would be “As far as the arithmetic value of the caterpillar  equals to 5, and in as much the values of bee, cowslip, and butterfly depends on the value of caterpillar, we can easily estimate the value of bee, cowslip and butterfly which is 10, 20 and 50 respectively.

Slightly after, the teacher asks the students to work in pairs and form the equations of the system presented within the task. The teacher reminds the students that they should remember to use the same variable every time that a specific icon appears e.g the variable  $x$  for the shirt, the variable  $y$  for the skirt, the variable  $z$  for the shoe and the variable  $n$  for the dress. Accordingly, the students come up with the following linear system of equations:

$$2x + y = 50 \text{ [equation 1]}$$

$$z + 2y = 70 \text{ [equation 2]}$$

$$y + 2n = 65 \text{ [equation 3]}$$

$$x + y = 35 \text{ [equation 4]}$$

Then, the teacher makes again an auxiliary question, as follows: “What will be our criterion in selecting the equations from which we are going to start?” This auxiliary question will spark the discussion and students will start exchanging opinions, to end up with the correct answer which is: “We should start working on the system of 2 equations and 2 variables which is formulated by the equation 1 and 4; those equations contain the same group of variables, namely x and y”

Then, the teacher asks students to work in pairs and try to solve on the linear system of equations:

$$2x + y = 50 \text{ [eq. 1]}$$

$$x + y = 35 \text{ [eq. 4]}$$

The solution of the aforementioned system is given below: As a first step we solve the equation 4 for y in terms of x:

$$x + y = 35$$

$$y = 35 - x$$

Then we substitute this expression of y into the other equation of the linear system [eq.1]. Hence the equation  $2x + y = 50$  will take the form of:  $2x + 35 - x = 50$

$$x + 35 = 50$$

$$x = 50 - 35$$

$$x = 15$$

By substituting the value of x into eq.1 or eq.4 we get:  $y = 20$

Accordingly, up to now we know that the value of shirt equals to 15, whilst the value of skirt equals to 20.

At this stage the teacher asks students to announce the values of x and y. Afterwards, he asks the students to work in pairs to end up with the final solution.

Students soon realise that since the value of y is known, they can substitute it in both the remaining equations to draw the following results:

$$z + 2y = 70 \text{ (eq. 2)}$$

$$z + 40 = 70$$

$$z = 30 \text{ (value of shoe)}$$

$$y + 2n = 65$$

$$20 + 2n = 65$$

$$2n = 65 - 20$$

$$2n = 45$$

$$n = 22.5 \text{ (value of skirt)}$$

The teacher asks students to announce the values of  $z$  and  $n$ , or in other words the values of shoe and dress.

## EVALUATION

### FINAL PART (5 minutes)

1. DO I KNOW the theory behind the linear systems?

2. DO I UNDERSTAND how to use the method of substitution to solve a linear system?

3. CAN I USE IT to solve:

1. the concrete system by the substitution method  $2 \times 2$ ?
2. a specific system by the method of  $3 \times 3$ ?

4. CAN I EXPLAIN IT to my classmates?

## INCLUSIVENESS GUIDELINES

Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.
- You can use checklists for your students to make sure they have done all the steps
- Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.
- At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process
- Make sure that the material the students manipulate is easy enough to grasp
- While using different media (paper, computer and visual aids) choose different background than white which can be too bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colors to learn more about student's preference.
- To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.

### EXAMPLE:

#### 1. Start every lesson with a short "CHECK-IN"

- Today, we will study the topic (name of the topic)
- I will tell you about: (name 3 keywords connected with the topic)
- Then I will present exercises: (name the exercises from the student book)
- Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)
- Once the exercises will be done [To continue]

#### 2. Then finish lesson with a short "CHECK-OUT"

- During the lesson we learn about (topic of the lesson)



- The most important things were: (name 3 keywords connected with the topic)
- We were able to do... (tell about the work student done during the lesson)
- We will explore the topic next time when we will learn about (name the following topic)

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.