LESSON SCENARIO 08:
THE ARITHMETIC MEAN - GEOMETRIC MEAN INEQUALITY

Topic: The Arithmetic Mean - Geometric Mean Inequality

Level: Age 14 - 15

Foreknowledge: operations with fractions, operations with radicals, arithmetic mean, geometric mean, initial capital, final capital, simple interest and compound interest

Correlation: Financial mathematics, Art, Architecture

Time: 50 minutes

LEARNING OUTCOMES

- Calculation of arithmetic and geometric means in practical, concrete situations

TEACHING METHODS

- VR technology
- Frontal work
- Individual work and pair work

KEY WORDS

- means
- arithmetic mean
- geometric mean
- inequality

RESOURCES

- VR headsets
- blackboard
- worksheets
- geometric instruments, scissors
- laptop/ computer, pocket calculator, projector

ACTIVITIES

INTRODUCTION: RULES OF CONDUCT WHEN USING VR IN THE CLASSROOM (5 min)

The teacher starts discussion with the students asking them about the use of VR and their expectations in using VR in classroom.

After the discussion, the teacher defines the work methods and rules of conduct for students regarding safety precautions for using VR headsets in the classroom and learning in virtual environment:

- **listen to the teacher carefully**
- **remove physical obstacles before using VR**
- **always work in pairs - never alone**
- **keep the device clean.**

ACTIVITY 1 (5 min) THE ARITHMETIC MEAN AND THE GEOMETRIC MEAN

Form of work: frontal

Required accessories: blackboard or prepared PowerPoint

The teacher presents the topic of the lesson and reminds the students the following concepts:
The word "average/mean" is found almost daily in people's discussions, in expressions such as: "average duration of people's life", "average life of a device", "average weight of a product". The average is a typical or central value of a lot of data. In order for the average size to have an objective character, it is necessary to choose the right type of mean (mathematical name for average). The most used means are: arithmetic mean; geometric mean; harmonic mean; sqare/quadratic mean.

The arithmetic mean of a list of *n* numbers $x_1, x_2, \dots, x_n$ is the sum of the numbers divided by *n*:

$$\frac{x_1 + x_2 + \cdots + x_n}{n}.$$

The geometric mean is defined as the *n* th root of the product of *n* non-negative numbers. For a set of *n* non-negative numbers $x_1, x_2, \dots, x_n$, the geometric mean is defined as:

$$\sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}.$$

ACTIVITY 2 (5 min) THE ARITHMETIC MEAN - GEOMETRIC MEAN INEQUALITY

Form of work: frontal

Required accessories: blackboard or prepared PowerPoint

The teacher presents the inequality of means.

For a list of *n* non-negative numbers $x_1, x_2, \ldots, x_n$, using mathematical notations, AM–GM, the inequality is written as:

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n},$$

and that equality holds if and only if $x_1 = x_2 = \cdots = x_n$.

The case for two non-negative numbers a and b, is the statement that

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

with equality if and only if $a = b$.

AM–GM inequality is a basic inequality, used to demonstrate other inequalities.

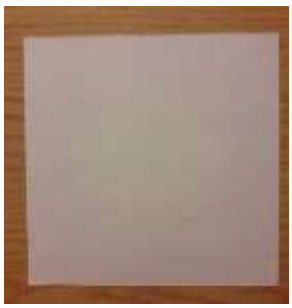
ACTIVITY 3 (10 min) A VISUAL AND A MATHEMATICAL DEMONSTRATION

The teacher dives the studens in pairs and explain the next steps.

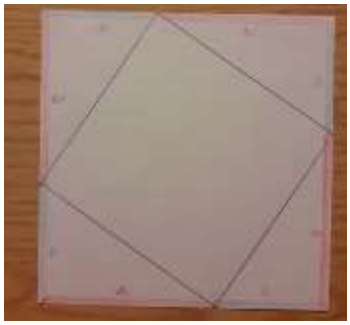
Form of work: work in pairs

Required accessories: worksheets, geometric instruments, scissors

Below you have a visual demonstration:

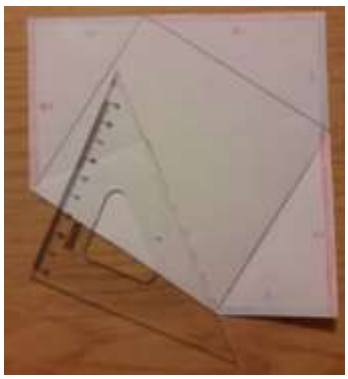


1. Make a square out of a paper sheet.



2. Divide every side into two segments with the length a and b.

3. Draw a line from one point to the other, as seen in the pictures.



4. Fold the piece of paper alongside the segments resulted.

5. Draw a dashed line alongside the longer side (of length a in our illustration).

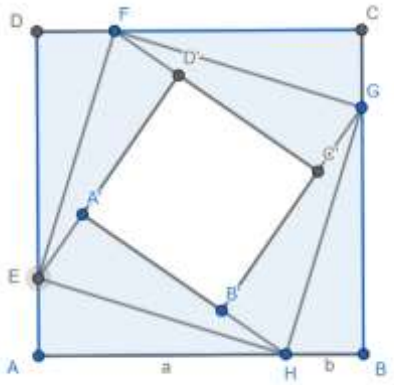
6. The area of the square with side $a+b$, $(a+b)^2$ is 8 times greater than the one of a right triangle with catheti a and b, which is $8 \cdot \frac{a \cdot b}{2}$. And we get the inequality, only if $a = b$.

Once the students finish the activity, the teacher presents an app called geobra.org, which models the same idea that the students have worked on. Then the teacher demonstrates the inequality (the classical way):

$$\frac{a+b}{2} \geq \sqrt{a \cdot b} \iff a+b \geq 2\sqrt{a \cdot b} \iff$$

$$(a+b)^2 \geq 4ab \iff a^2+2ab+b^2 \geq 4ab \iff$$

$$a^2-2ab+b^2 \geq 0 \iff (a-b)^2 \geq 0.$$



ACTIVITY 4 (10 min) INEQUALITY OF MEANS IN VR APPLICATION

The teacher assigns the task to the students.

The student:

- **finds and selects the INEQUALITY OF MEANS exercise on the exercise shelf**
- **solves tasks in VR application**
- **the student will modify the size of the segments a and b observing the differences between the two surfaces**

Form of work: work in pairs

Required accessories: VR headset

COURSE OF ACTIVITY:

The teacher divides the students into pairs. Student A carefully puts on his VR headset, opens the INEQUALITY OF MEANS exercise in a virtual library in a VR application and solves the tasks in a virtual environment. Student B assists him/her. This is followed by a swap of roles.

ACTIVITY 5 (15 min) Worksheet

Complete solutions are required for all the problems.
The use of the pocket computer is allowed.

1. Determine the value of truth of the following sentence:
"$\frac{a+b}{2} > \sqrt{ab}, \forall a, b \in (0, \infty)$**" (true or false) and explain the choice you made.**

2. Calculate the arithmetic mean of the numbers 3, 4, 27, 64. Calculate the geometric mean of the numbers: 3, 4, 27, 64. Compare the results.

3. Calculate the artihmetical mean of the numbers: $3 + \sqrt{8}$ **and** $3 - \sqrt{8}$**. Calculate the geometric mean of the numbers:** $3 + \sqrt{8}$ **and** $3 - \sqrt{8}$**. Compare the results.**

Work sheet solutions:
1. The statement is false. If a = b, then the inequality becomes equality.

2. AM= $\frac{3+4+27+64}{4} = \frac{98}{4} = 24,5$**. GM=** $\sqrt[4]{3 \cdot 4 \cdot 27 \cdot 64} = \sqrt[4]{2^8 \cdot 3^4} = 2^2 \cdot 3 = 12$**.**
AM>GM.
3. AM= $\frac{(3+\sqrt{8})+(3-\sqrt{8})}{2} = \frac{6}{2} = 3$**. GM=** $\sqrt{(3+\sqrt{8})(3-\sqrt{8})} = \sqrt{9-8} = 1$**. AM>GM.**

EVALUATION

1. I like the way of work in this lesson	1	2	3	4	5
2. This lesson was interesting	1	2	3	4	5
3. It is clear what I was supposed to learn in this lesson	1	2	3	4	5
4. The subject matter was clearly explained	1	2	3	4	5
5. I have learned the subject matter	1	2	3	4	5
6. I think I actively participated in this lesson	1	2	3	4	5
7. I was more active in this lesson than usually	1	2	3	4	5
8. By being active I contributed to the quality of the lesson	1	2	3	4	5
9. I was motivated for work in this lesson	1	2	3	4	5
10. I prefer using VR in lessons	1	2	3	4	5
111. Name two things you liked in this lesson:					
12. Name two things you didn`t like in this lesson:					

INCLUSIVENESS GUIDENESS

Every student is different and their needs for the material might vary. Below you will find several tips that could make mathematics lesson more inclusive for students who struggle with learning disorders.

- **When giving assignments to classroom try to break them into small pieces of information. Avoid the double tasks in the instructions. Remember that in case of operations/exercises with multiple steps, it is critical to help learners decompose the steps.**
- **You can use checklists for your students to make sure they have done all the steps**
- **Make sure the font, line spacing, and alignment of your document is accessible for students with learning disorders. It is recommended to use a plain, evenly spaced sans serif font such as Arial and Comic Sans. Others: Verdana, Tahoma, Century Gothic and Trebuchet. Spacing should be 1.5 and try to avoid justification in the text.**
- **At the end of each activity, take some time to ask the students what they have learnt to acknowledge every step in their learning process**
- **Make sure that the material the students manipulate is easy enough to grasp**
- **While using different media (paper, computer and visual aids) choose different background than white which can be too bright for students with learning disorders. The best choice would be cream or soft pastel but try to test different colours to learn more about student's preference.**
- **To stimulate short and long-term memory prepare for all the students in the classroom an outline describing what they are going to learn on this lesson and finish it with a resume of what has been taught. In this way they will strengthen the ability to remember information.**

EXAMPLE:

1. Start every lesson with a short "CHECK-IN"

- **Today, we will study the topic (name of the topic)**
- **I will tell you about: (name 3 keywords connected with the topic)**
- **Then I will present exercises: (name the exercises form the student book)**
- **Then we will do exercises (explain the way student will be working: ex. together with teacher / in pairs /individually)**
- **Once the exercises will be done [To continue]**

2. Then finish lesson with a short "CHECK-OUT"

- **During the lesson we learn about (topic of the lesson)**

- **The most important things were: (name 3 keywords connected with the topic)**
- **We were able to do… (tell about the work student done during the lesson)**
- **We will explore the topic next time when we will learn about (name the following topic)**

It is a small adjustment that will take 5 min from the lesson but can make a great difference in the way that the material will be remembered. Try to create this as a routine habit.